UNIT – I
Resonance

Objectives:

- To understand the concepts of Resonance, Bandwidth and Quality factor.
- To evaluate Resonance, Bandwidth and Quality factor for various series parallel combinations of R, L, C.

Syllabus:

Resonance - series, parallel circuits, concept of band width and Q factor.

Outcomes:

On completion the student should be able to:

- understand the concept of electrical resonance, Bandwidth and Quality factor
- Evaluate resonant frequency, Bandwidth and Quality factor for various series and parallel RLC circuits.

Resonance:

Resonance is a particular type of phenomenon inherently found normally in every kind of system, Electrical, Mechanical, Optical, Acoustical and even Atomic. Usually resonance occurs in any of these systems, when energy storage elements interchange exactly equal amounts of energy. Resonance cannot take place when only one type of energy storing element is present such as an inductance or a mass. There must exist two types of independent energy storing elements capable of interchanging energy between one another, i.e., inductance and capacitance in electrical systems and mass and a spring in mechanical systems.

There are several definitions of resonance. But, the most frequently used definition of resonance in electrical systems is the steady state operation of a circuit or system at that frequency for which the resultant response is in time with the exciting function, despite the presence of energy storing elements. Resonance is a phenomenon which enables us to discriminate between different frequencies. Using resonant circuits, it is possible to select a particular frequency from a band of frequencies.

Series Resonance

A circuit is said to be under resonance, when the applied voltage V and the resulting current I are in phase. Thus a series RLC circuit, under resonance behaves like a pure resistance network and the net reactance of the circuit should be zero. Since V and I are in phase, the p.f. is unity at resonance.

Consider the series RLC circuits shown in figure 1.1.

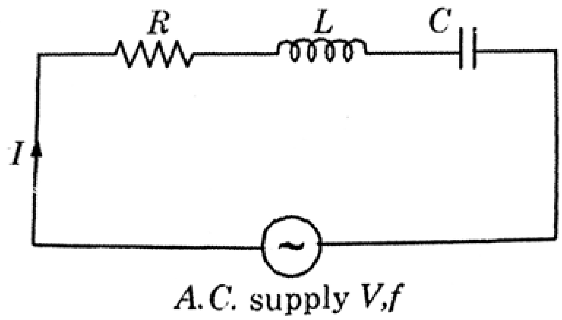


Figure 1.1

The complex impedance of the circuit is

$$Z = R + j(X_L - X_C) \Rightarrow Z = R + j\left\{\omega L - \left(\frac{1}{\omega C}\right)\right\}$$

The absolute value of impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Under resonance, the circuit should be purely resistive, i.e., net reactance should be zero.

$$X = X_L - X_C = \omega L - \left(\frac{1}{\omega C}\right) = 0 \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega = \left(\frac{1}{\sqrt{LC}}\right) \text{ or } f = \left(\frac{1}{2\pi\sqrt{LC}}\right) \Rightarrow \omega_0 = 2\pi f_0 = \left(\frac{1}{\sqrt{LC}}\right)$$

is called the natural frequency of the circuit.

We can obtain resonance by changing the frequency of the applied voltage. When it is equal to $f = \frac{1}{2\pi\sqrt{LC}}$, i.e., the natural frequency of the circuit, it is under resonance and this frequency called resonant frequency. Resonance can also be obtained by varying L or C. The general considerations at resonance are the same regardless of which parameter is varied to produce resonance.

Behaviour of RLC series circuit under Variable Frequency

From equation $Z = R + j\left\{\omega L - \left(\frac{1}{\omega C}\right)\right\}$ we see that only imaginary part of the impedance is a function of frequency. The components of Z, R, X_L and X_C, how they vary with frequency are shown in figure 1.2.

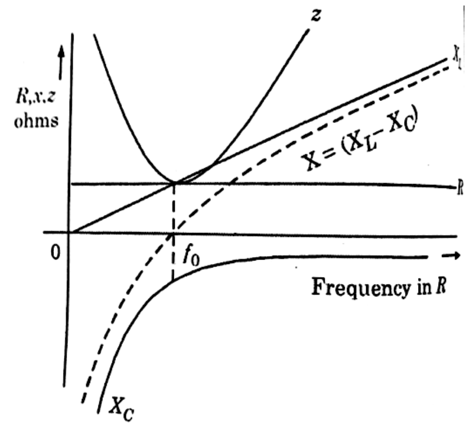


Figure 1.2

1. **Resistance R** is independent of frequency.
2. **Inductive reactance X_L** directly proportional to frequency and is positive.
3. **Capacitive reactance X_C** Inversely proportional to frequency and is negative.
4. **Net reactance $X = X_L - X_C$** It will be initially negative and becomes zero and then it is positive.

i.e. for $f < f_0$, it is capacitive

for $f = f_0$, it is zero

for $f > f_0$, it is inductive.

When, $X_L = X_C$ the circuit is said to be under resonance.

5. Impedance (Z)

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L - X_C = 0$, hence $Z = R$. At any other frequency, $(X_L - X_C) \neq 0$ and hence $|Z| > R$.
Hence, impedance at resonance is minimum and it is equal to R. The circuit behaves as capacitive circuit below f_0 and inductive circuit f_0.

Current at Resonance

The current= $\frac{\text{Voltage}}{\text{Impedance}} = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$ since the impedance is minimum and equal to R at resonance, the current is maximum and equal to $\left(\frac{V}{R}\right)$ and in phase with V (power factor is unity). It varies inversely as impedance Z. The variation of current with frequency is shown in fig 1.3.

$$I_{max} = \frac{V}{R}$$

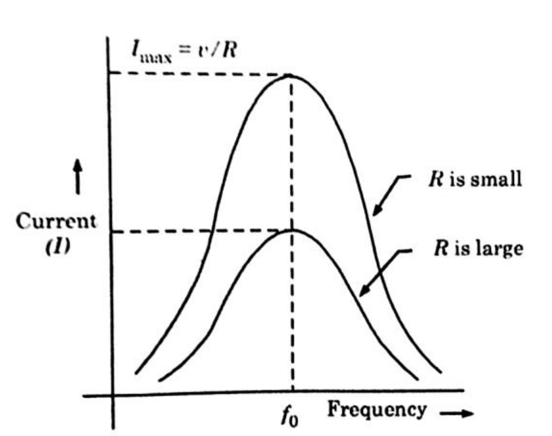


Figure 1.3

With all parameters being same, if R is varied the current at resonance will change but resonance frequency is independent of R. The shape of current variation becomes flat as resistance is increased.

Since current is maximum at resonance, voltage across resistance, V_R = IR will also be maximum and equal to applied voltage.

Voltage across Elements R, L and C

Voltage across resistance R=IR is maximum at resonance and equal to voltage applied to the series circuit.

Voltage across inductive reactance = IX_L.

Both I and X_L are increasing before resonance and the product must be increasing. At resonance, I is not changing but X_L is increasing and hence the product should be increasing, the voltage across inductor continues to increase until the reduction in current offsets the increase in X_L.

IX_L is maximum after f_0.

In the case of voltage across capacitor $V_C = IX_C$. At resonance, I is constant and X_C is decreasing. Therefore, the product should be decreasing. Hence IX_C should have been maximum before resonance frequency f_0. The variation of voltage V_R, V_L and V_C are shown in figure 1.4.

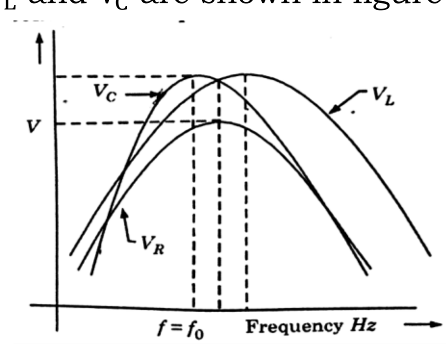


Figure 1.4

Frequency at which V_L is maximum

We know that

$$V_L = IX_L = \frac{V\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_L{}^2 = \frac{V^2\omega^2L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{V^2\omega^4L^2C^2}{\omega^2C^2R^2 + (\omega L2C - 1)^2}$$

To determine frequency at which V_L is maximum, we will equate $\frac{d(V_L{}^2)}{d\omega} = 0$

$$\frac{4\omega^3V^2L^2C^2[\omega^2C^2R^2 + (\omega^2LC - 1)^2] - V^2\omega^4L^2C^2[2\omega C^2R^2 + 2(\omega^2LC - 1)2\omega LC]}{[\omega^2C^2R^2 + (\omega^2LC - 1)^2]^2} = 0$$

$$4\omega^3V^2L^2C^2[\omega^2C^2R^2 + (\omega^2LC - 1)^2] - V^2\omega^4L^2C^2[2\omega C^2R^2 + 2(\omega^2LC - 1)2\omega LC] = 0$$

$$2[\omega^2C^2R^2 + (\omega^2LC - 1)^2] - \omega[\omega C^2R^2 + 2\omega LC(\omega^2LC - 1)] = 0$$

$$2\omega^2C^2R^2 + 2(\omega^2LC - 1)^2 - \omega^2C^2R^2 - 2\omega^2LC(\omega^2LC - 1) = 0$$

$$\omega^2C^2R^2 + (\omega^2LC - 1)[2(\omega^2LC - 1) - 2\omega^2LC] = 0$$

$$\omega^2C^2R^2 - 2\omega^2LC + 2 = 0$$

$$\omega^2[2LC - C^2R^2] = 2$$

$$\omega = \frac{1}{\sqrt{LC - \frac{C^2R^2}{2}}}$$

$$f = \frac{1}{2\pi\sqrt{LC - \frac{C^2R^2}{2}}}$$

Frequency at which V_C is maximum

We know that

$$V_C = IX_C = \frac{V}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2}}\frac{1}{\omega C}$$

$$V_C{}^2 = \frac{V^2}{\omega^2C^2R^2 + (\omega^2LC - 1)^2}$$

To determine the frequency at which V_C is maximum we will equate $\frac{d(V_C{}^2)}{d\omega} = 0$

$$\frac{-V^2[2\omega C^2R^2 + 2(\omega^2LC - 1)2\omega LC]}{[\omega^2C^2R^2 + (\omega^2LC - 1)^2]^2} = 0$$

$$[2\omega C^2R^2 + 2(\omega^2LC - 1)2\omega LC] = 0$$

$$[2\omega C^2R^2 + 4\omega LC(\omega^2LC - 1)] = 0$$

$$2\omega C[CR^2 + 2L(\omega^2LC - 1)] = 0$$

$$CR^2 + 2L^2\omega^2 C - 2L = 0$$

$$2L^2\omega^2 C = 2L - CR^2$$

$$\omega^2 = \frac{2L - CR^2}{2L^2 C} = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \text{ or } f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

At resonance the conditions are

1. Circuit is purely resistive
2. Power factor of the circuit is unity
3. Current is maximum and is equal to $\frac{V}{R}$
4. Voltage across L= Voltage across C
5. The p.f of the circuit changes from leading to lagging with increase in frequency.

Series RLC circuit as a frequency selector

Any series RLC circuit passes all waves of finite frequency to some extent but it will offer lowest impedance at resonant frequency i.e., it allows frequencies near the resonant frequency more rapidly than other frequencies. Here, a series RLC circuit posses the frequency selectivity, i.e., the ability to discriminate among waves of different frequencies. The band of frequencies which is passes quite readily through the circuit is called Pass band or Band width of the circuit.

Bandwidth or Pass band It is arbitrarily considered to be the range of frequencies over which the current is equal to the greater than $\frac{1}{\sqrt{2}}$ times the current at resonance $\left(= \frac{V}{R}\right)$.

It is customary to specify two frequencies f_1 and f_2 at which the current is reduced to $\left(\frac{V}{\sqrt{2}R}\right)$ as shown in figure 1.5.

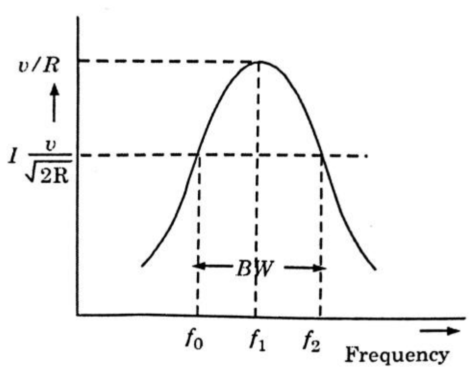


Figure 1.5

Within this range of frequencies the power dissipation is equal to or greater than $\frac{V^2}{2R}$, i.e., half of the maximum power at resonance, $\frac{V^2}{R}$ and hence the frequencies f_1 and f_2 are called lower and upper half power frequencies. The frequency range $f_2 - f_1$ is called Bandwidth or Pass band.

$$\text{Bandwidth} = f_2 - f_1$$

Determination of Bandwidth

Current at resonance = $\frac{V}{R}$

Impedance at resonance = R

Current at half power frequencies f_1 and f_2 is $\frac{1}{\sqrt{2}}$ times the current at resonance

$$I = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{V}{R}\right)$$

Therefore, impedance Z at half power frequencies

$$= \sqrt{2}R$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{2}R$$

Squaring, $R^2 + X^2 = 2R^2 \Longrightarrow X^2 = R^2 \Longrightarrow X = R$

Hence, at frequencies f_1 and f_2 the net reactance is equal to the resistance, i.e., $X = R$. The phase angle at these frequencies will be 45^0 and hence power factor is 0.707.

At lower half power frequency, f_1, $X_C > X_L$, hence, the net reactance

$$X = \left(\frac{1}{\omega_1 C}\right) - \omega_1 L = R$$

$$1 - \omega_1{}^2 LC = \omega_1 CR$$

Therefore,

$$\omega_1{}^2 LC + \omega_1 CR - 1 = 0 \Longrightarrow \omega_1{}^2 + \omega_1\left(\frac{R}{L}\right) - \left(\frac{1}{LC}\right) = 0$$

$$\omega_1 = -\left(\frac{R}{2L}\right) \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

At upper half frequency, f_2, $X_L > X_C$, hence

$$X = \omega_2 L - \left(\frac{1}{\omega_2 C}\right) = R$$

$$\omega_2{}^2 LC - 1 = \omega_2 CR \Longrightarrow \omega_2{}^2 - \omega_2\left(\frac{R}{L}\right) - \left(\frac{1}{LC}\right) = 0$$

$$\omega_2 = \left(\frac{R}{2L}\right) \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

Therefore, bandwidth= $\omega_2 - \omega_1 = \frac{R}{L} \Longrightarrow f_2 - f_1 = \frac{R}{2\pi L}$

Quantity Factor (Q-Factor)

The quantity of series resonant circuit depends on the sharpness of the current variation with frequency (current response). Smaller the value of resistance R compared to X_L and X_C, sharper will be the response. To introduce a quantitative measure for the quality of the resonant circuit, quality factor is defined as

$$Q-\text{factor}=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 CR}$$

$$=\frac{X_{L0}}{R}=\frac{X_{C0}}{R}$$

$$=\frac{\text{Inductive or capacitive reactance at resonance}}{\text{Resistance}}$$

It can also defined as the ratio of voltage across inductor or capacitor at resonance to the supply voltage

$$Q-\text{factor}=\left(\frac{V_L}{V}\right)=\left(\frac{V_C}{V}\right)$$

Since $\quad V_L=\omega_0 LI$

and $\quad V=IR$

$$Q-\text{factor}=\frac{V_L}{V}=\frac{\omega_0 LI}{RI}=\frac{\omega_0 L}{R}$$

Another form of quality factor in terms of energy is defined as

$$Q=2\pi\left(\frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}\right)$$

The maximum energy stored in inductor at resonance is

$$W_L=\left(\frac{1}{2}\right)L{I_m}^2$$

Power dissipated at resonance

$$P_0=\frac{{I_m}^2 R}{2}$$

Power dissipation per cycle $=\frac{P_0}{f_0}$

$$Q=\frac{\left[2\pi\left(\frac{1}{2}\right)L{I_m}^2\right]}{\left[\frac{{I_m}^2 R}{2f_0}\right]}=2\pi\left(\frac{1}{2}\right)L{I_m}^2.\frac{2f_0}{{I_m}^2 R}=\frac{2\pi f_0 L}{R}=\frac{\omega_0 L}{R}$$

Selectivity

The ratio of bandwidth to resonance frequency is defined as selectivity of the circuit

$$\text{Selectivity}=\frac{BW}{f_0}=\frac{f_2-f_1}{f_0}=\frac{\frac{R}{2\pi L}}{\frac{1}{2\pi\sqrt{LC}}}=\frac{R}{L}\sqrt{LC}=R\sqrt{\frac{C}{L}}=\sqrt{\frac{CR^2}{L}}$$

Parallel Resonance

The general definition of resonance that the circuit containing energy storage elements (L and C) behaves as a pure resistive network at resonance and the applied voltage and resulting current are in phase, is also applicable to parallel resonance. At resonance, in a parallel circuit, the net susceptance must be zero. In analyzing series circuit we have employed impedance concept and for parallel circuit admittance concept is more convenient. First we consider a parallel circuit with ideal elements (R, L and C) in each branch and then extend the conditions to a general circuit.

Parallel resonance in pure RLC circuits

Let us consider a parallel circuit in which each branch consists of single ideal element (R, L and C) as shown in figure 1.6 as

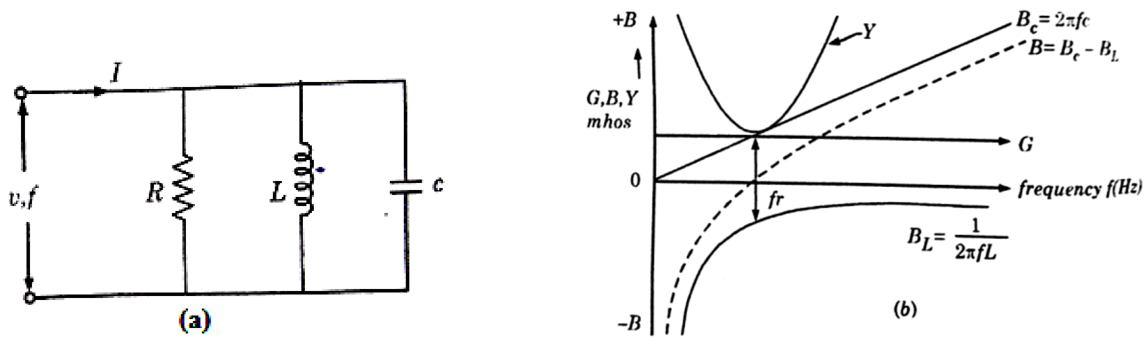


Figure 1.6

The admittances of each branch (reciprocal of impedances) are

$$Y_1 = \frac{1}{R} = G$$

$$Y_2 = \frac{1}{jX_L} = -jBL = -j\frac{1}{\omega L}$$

$$Y_3 = \frac{1}{jX_C} = jB = j\omega C$$

The total admittance of the circuit is

$$Y = Y_1 + Y_2 + Y_3 = G + j(B_C - B_L)$$

The condition for resonance is that net susceptance should be zero

$$B_C - B_L = 0 \Longrightarrow B_C = B_L$$

$$\omega C = \frac{1}{\omega L}$$

The frequency at resonance is

$$\omega_r = \frac{1}{\sqrt{LC}} \Longrightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

The absolute value of total admittance is $|Y| = \sqrt{G^2 + (B_C - B_L)^2}$. At $f = f_r$, total admittance $|Y| = G$ only. Since net susceptance is zero it is also minimum, the p.f is unity. At $f > f_r$ the net susceptance is capacitive and hence the p.f of the circuit is leading. The effect of variation of frequency on conductance, susceptance and admittance are shown in figure 1.6 b).

Variation of Total current

$$I = \frac{V}{Z} VY$$

Since admittance is minimum at resonance, the current is also minimum. The variation of total current with frequency is same as the variation of admittance and is shown in figure 1.7. Sometimes a parallel resonance circuit is called anti resonant circuit since current is minimum.

Current at resonance $I_0 = VG$

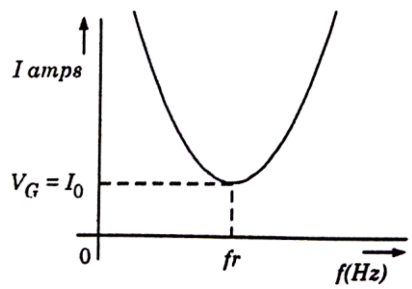


Figure 1.7

Conditions at Resonance in parallel circuits

1. The inductive and capacitive susceptance are equal
2. Net susceptance is zero
3. Admittance is minimum and equal to conductance
4. The current is minimum and is equal to VG
5. The V and I are in phase and power factor is unity.

But in practice it is not necessary to have a separate resistance branch, because the inductive and capacitance are always associated with small resistances.

Practical two branch resonant circuit

In a practical resonant circuit shown in figure 1.8 a) and inductance and capacitance elements are connected in parallel and having resistance associated with them. The phasor diagram showing the applied voltage, branch currents and total current are shown in figure 4.13 b) at resonance condition.

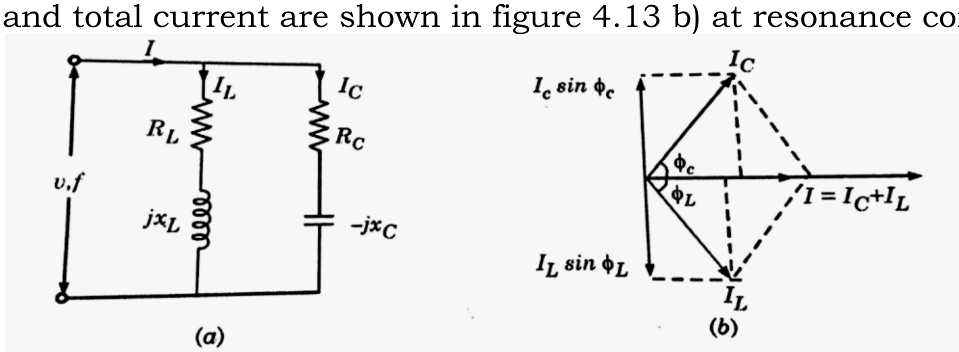


Figure 1.8

In order that the above circuit is under resonance, the total current should be in phase with the applied voltage, i.e. it should behave as a pure resistive circuit. In order that the total current is in phase with applied voltage the net reactive component of current should be zero.

i.e., $I_L \sin \phi_L = I_C \sin \phi_C$

$$\frac{V}{\sqrt{R_L^2+X_L^2}} \frac{X_L}{\sqrt{R_L^2+X_L^2}} = \frac{V}{\sqrt{R_C^2+X_C^2}} \frac{X_C}{\sqrt{R_C^2+X_C^2}}$$

$$\frac{X_L}{R_L^2+X_L^2} = \frac{X_C}{R_C^2+X_C^2}$$

i.e., Inductive susceptance = Capacitive susceptance

Net susceptance is zero. Hence the admittance at resonance is pure conductance and is equal to

$$Y = \frac{R_L}{R_L^2+X_L^2} + \frac{R_C}{R_C^2+X_C^2}$$

The condition for resonance is

$$\frac{X_L}{R_L^2+X_L^2} = \frac{X_C}{R_C^2+X_C^2} \Longrightarrow \frac{\omega_r L}{R_L^2+\omega_r^2 L^2} = \frac{\frac{1}{\omega_r C}}{\left(R_C^2+\frac{1}{\omega_r^2 C^2}\right)}$$

Cross multiplying

$$\omega_r^2 LC\left(R_C^2 + \frac{1}{\omega_r^2 C^2}\right) = R_L^2 + \omega_r^2 L^2$$

$$\omega_r^2 LC\left[R_C^2 - \frac{L}{C}\right] = R_L^2 - \frac{L}{C}$$

$$\omega_r = \frac{1}{\sqrt{LC}}\left[\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}}\right]^{1/2}$$

Resonance frequency

$$f_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\left[\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}}\right]^{1/2}$$

Since the resonance frequency f_r must be a real value, it is necessary that both the quantities $\left(R_L^2 - \frac{L}{C}\right)$ and $\left(R_C^2 - \frac{L}{C}\right)$ should be of the same sign (either +ve or -ve). If this is not satisfied resonance will not occur.

Since the circuit behaves as a pure conductance and hence the current at resonance is

$$I_0 = VG = V\left[\frac{R_L}{R_L^2+X_L^2} + \frac{R_C}{R_C^2+X_C^2}\right]$$

Usually the resistance associated with capacitance is small and considering an ideal capacitor i.e. $R_C = 0$, then

Resonance frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}\left[\sqrt{\frac{{R_L}^2-\frac{L}{C}}{-\frac{L}{C}}}\right]^{1/2}$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\left(\frac{{R_L}^2}{L^2}\right)}$$

Current at resonance $I_0 = \frac{VR_L}{{R_L}^2+{X_L}^2}$

Impedance at resonance $= \frac{{R_L}^2+{X_L}^2}{R_L} = \frac{{R_L}^2+{\omega_r}^2L^2}{R_L} = R_L + \frac{L}{CR_L} - R_L = \frac{L}{CR_L}$

The impedance at resonance $\frac{L}{CR_L}$ is called dynamic resistance or effective resistance of the circuit.

Resonance can also be obtained by varying different parameters in the above circuit.

Resonance by varying L and keeping all other parameters constant

In the parallel circuit shown in fig 1.8 a), we would like to get resonance at $\omega = \omega_r$ by varying only X_L. The conditions for resonance is

$$\frac{X_L}{{R_L}^2+{X_L}^2} = \frac{X_C}{{R_C}^2+{X_C}^2}$$

Cross multiplying we will get

$$X_L({R_C}^2+{X_C}^2) = X_C({R_L}^2+{X_L}^2)$$

The above equation is written as a quadratic equation in the variable X_L

$$X_C{X_L}^2 - X_L({R_C}^2+{X_C}^2) + X_C{R_L}^2 = 0$$

Solving for X_L

$$X_L = \omega_r L = \frac{\left[({R_C}^2+{X_C}^2)\pm\sqrt{({R_C}^2+{X_C}^2)^2-4{X_C}^2{R_L}^2}\right]}{2X_C}$$

$$\omega_r L = \frac{{Z_C}^2\pm\sqrt{{Z_C}^4-4{X_C}^2{R_L}^2}}{2X_C}$$

$$\omega_r L = \frac{{Z_C}^2\pm\sqrt{{Z_C}^4-4{X_C}^2{R_L}^2}}{\frac{2}{\omega_r C}}$$

$$L = \frac{C}{2}\left[{Z_C}^2 \pm \sqrt{{Z_C}^4 - 4{X_C}^2{R_L}^2}\right]$$

In similar way, resonance can be obtained by varying by varying C (or X_C), R_L or R_C in the above circuit. The corresponding values ofX_C,R_L or R_C can be obtained from equation$X_L({R_C}^2+{X_C}^2) = X_C({R_L}^2+{X_L}^2)$.

The value of C at resonance $C = \frac{2L}{\left({Z_L}^2\pm\sqrt{{Z_L}^4-4{R_C}^2{X_L}^2}\right)}$

Where $Z_L = \sqrt{{R_L}^2+{X_L}^2}$

The value of RL at resonance $\mathrm{R_L} = \sqrt{\omega_r^2 + \mathrm{LCR_C}^2 - \omega_r^2 \mathrm{L}^2 + \frac{\mathrm{L}}{\mathrm{C}}}$

The value of RL at resonance $\mathrm{R_C} = \sqrt{\frac{\mathrm{R_L}^2}{\omega_r^2 \mathrm{LC}} + \frac{\mathrm{L}}{\mathrm{C}} - \frac{1}{\omega_r^2 \mathrm{C}^2}}$

Resonance at all Frequencies

Now, we would like to obtain the condition so that the two branch parallel circuit resonates at all frequencies. The condition for resonance is

$$\frac{\mathrm{X_L}}{\mathrm{R_L}^2+\mathrm{X_L}^2} = \frac{\mathrm{X_C}}{\mathrm{R_C}^2+\mathrm{X_C}^2}$$

$$\frac{\omega L}{\mathrm{R_L}^2+\omega^2 L^2} = \frac{\omega \mathrm{C}}{1+\omega^2 \mathrm{C}^2 \mathrm{R_C}^2}$$

$$\frac{1}{\left(\frac{\mathrm{R_L}^2}{\mathrm{L}}\right)+\omega^2 L} = \frac{1}{\frac{1}{\mathrm{C}}+\omega^2 \mathrm{CR_C}^2}$$

In order that the above equation is independent of ω we must have

$$\frac{\mathrm{R_L}^2}{\mathrm{L}} = \frac{1}{\mathrm{C}} \text{ i.e., } \mathrm{R_L}^2 = \frac{\mathrm{L}}{\mathrm{C}}$$

$$\mathrm{L} = \mathrm{CR_C}^2 \mathrm{R_C}^2 = \frac{\mathrm{L}}{\mathrm{C}}$$

Hence the condition for resonance at all frequencies is

$$\mathrm{R_L}^2 = \mathrm{R_C}^2 = \frac{\mathrm{L}}{\mathrm{C}}$$

$$\boxed{\mathrm{R_L} = \mathrm{R_C} = \sqrt{\frac{\mathrm{L}}{\mathrm{C}}}}$$

Q-Factor of Resonant Circuit

The parallel circuit behaves as a pure conductance at resonance and has minimum admittance maximum impedance. The circuit offers very high impedance for frequencies near resonance and hence does not allow these frequencies readily than other frequencies, i.e., the circuit will reject the frequencies near the resonant frequency and hence this resonance is called Rejector resonance.

In this circuit, at resonance total current I is minimum and the branch current I_L and I_C will equal and much greater than the total current.

The ratio of $\frac{I_L \text{ or } I_C}{I}$ at resonance is called current magnification.

The above ratio also represents quality factor or Q-factor.

The quality factor is also given by $\frac{B_L \text{ or } B_C}{G}$ at resonance

$$\mathrm{Q-factor} = \frac{B_L \text{ or } B_C}{G} \quad at\ resonance$$

UNIT - II

Balanced Three Phase circuits

Objectives:

- To introduce the concept of three phase electrical supply.
- To analyze three phase balanced systems
- To Measure three phase active and reactive power.

Syllabus:

Phase sequence-Star and Delta connection-Relation between line and phase voltages and currents in balanced systems-Analysis of balanced three phase circuits-Analysis -Star Delta transformation Technique-Measurement of Active and Reactive power in balanced three phase systems-Two wattmeter method of measurement of three phase power.

Outcomes:

On completion the student should be able to:

- Describe the reasons for, and the generation of the three-phase supply.
- Distinguish between star (3 and 4-wire) and delta connections.
- State the relative advantages of three-phase systems compared with single-phase-systems.
- Solve three-phase circuits in terms of phase and line quantities, and the power developed in three-phase balanced loads.
- Measure power dissipation in balanced three-phase loads, using the 1, 2 and 3-wattmeter methods, and hence determine load power factor.

1.1 Introduction:

There are two types of systems available in electrical circuits, single phase and three phases. In single phase circuits, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit.

- In 1882, new invention has been done called polyphase system, that more than one phase can be used for generating, transmitting and for load system.
- Three phase circuit is the polyphase system where three phases are sent together from generator to the load.

- Each phase are having a phase difference of 120^0, i.e 120^0 angle electrically. So from the total of 360^0, three phases are equally divided into 120^0 each.

  The sinusoidal waves for 3 phase system are shown below.

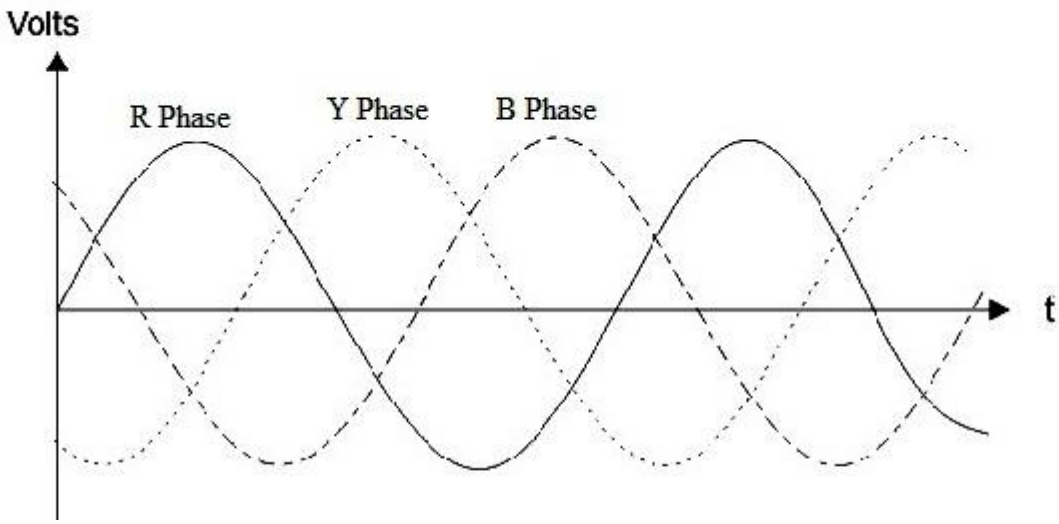


Fig.1.1 Three Phase Voltages

- The three phase can be used as three individual single phases. So if the load is single phase, then one phase can be taken from the three phase circuit and the neutral can be used as ground to complete the circuit.

1.1.1 Why three phase is preferred over single phase?

- There are number of advantages over single phase circuit.
- The three phase system can be used as three single phase line so it can act as three single phase system.
- The three phase generation and single phase generation is same in the generator except the arrangement of coil in the generator to get 120^0 phase difference.
- The conductor needed in three phase circuit is 75% that of conductor needed in single phase circuit.

A 3-phase system has the following advantages over single phase system.

- For a given frame size of a machine a 3-phase machine will have large capacity than a single phase machine.
- The torque produced in a 3-phase motor will be more uniform where as in a 1-phase motor it is pulsating.
- The amount of copper required in a certain amount of power over a particular distance, is less compared to a single phase system.

1.1.2 Phase sequence:

- It is the order in which the phase voltages will attain their maximum values.
- From the fig 1.1 it is seen that the voltage in R phase will attain maximum value first and followed by Y and B phases. Hence three phase sequence is RYB.
- This is also evident from phasor diagram in which the phasors with its positive direction of anti-clockwise rotation passes a fixed point is the order RYB, YBR and so on.
- The phase sequence depends on the direction of rotation of the coils in the magnetic field.
- If the coils rotate in the opposite direction then the phase voltages attains maximum value in the order RBY. The phase sequence gets reversed with direction of rotation.

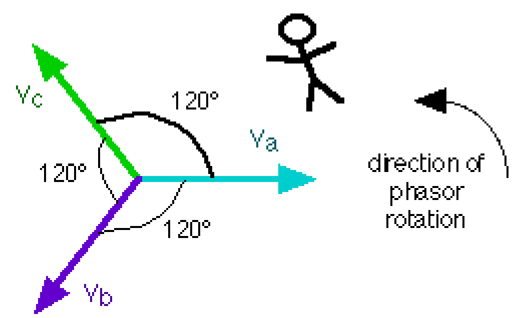


Fig.1.2 Phasor Representation of Three Phase Voltages

Then the voltage for this sequence can be represented as

$$v_R = v_m sin\ \omega t$$

$$v_Y = v_m \sin(\ \omega t - 120^0)$$

$$v_B = v_m \sin(\ \omega t - 240^0) = v_m \sin(\ \omega t + 120^0)$$

The RMS values of voltage can be expressed as

$$V_R = V_m \angle 0^0$$

$$V_Y = V_m \angle -120^0$$

$$V_B = V_m \angle -240^0 = V_m \angle +120^0$$

1.1.3 Star and Delta connection

- The three phase windings have six terminals i.e., R,Y,B are starting end of the windings and R',Y' and B' are finishing ends of windings.
- For 3 phase systems two types of common interconnections are employed.

1.1.3(a) Star connection:

- The finishing ends or starting ends of the three phase windings are connected to a common point as shown in. R', Y', B' are connected to a common point called neutral point.
- The other ends R, Y, B are called line terminals and the common terminal neutral are brought outside.
- Then it is called a 3 phase 4 wire star connected system.
- If neutral point is not available, then it is called 3 phase 3 wire star connection.

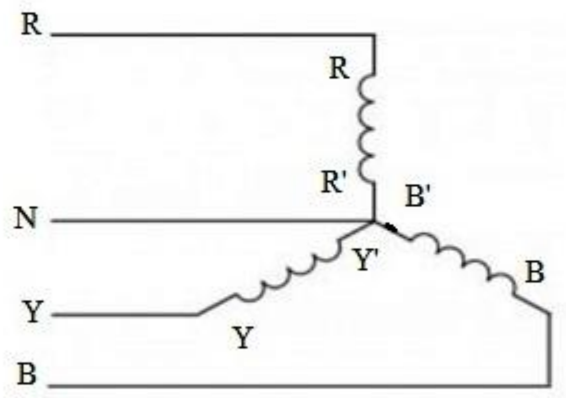


Fig.1.3 Star Connection

1.1.3(b) Delta connection:

- Here the dissimilar ends of the three coils i.e R and Y', Y and B', and B and R' are connected to form a closed Δ circuit (starting end of one phase is connected to finishing end of the next phase).
- The three ends are brought outside as line terminal R, Y, B. Three phase windings are connected in series and form a closed path.
- The sum of the voltages in the closed path for balanced system of voltages at any instant will be zero.

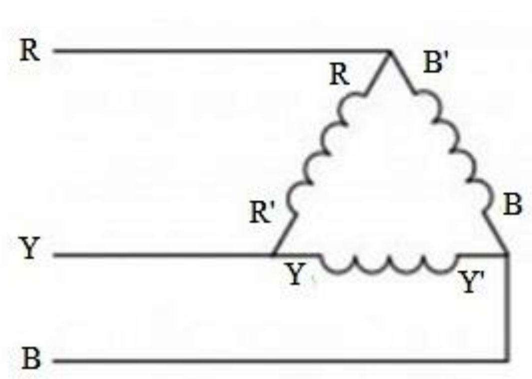


Fig.1.4 Delta Connection

- The main advantage of star connection is that we can have two different 3-phase voltages.
- The voltages between R & Y, Y & B, and B & R are called line voltages and form a balanced three phase voltage.
- The voltages between the terminals R & N, Y & N, and B & N are called phase voltage and form another balanced three phase voltage (line to neutral voltage or wye voltage).

1.2 Relation between line and phase voltage and currents in balanced systems:

In this section we will derive the relation between line and phase values of voltages and currents of 3-phase star connected and delta connected systems.

1.2.1 Star connection:

- Here, we employ double subscript notation to represent voltages and currents.
- The terminal corresponding to first subscript is assumed to be at a higher potential with respect to the terminal corresponding to second subscript.

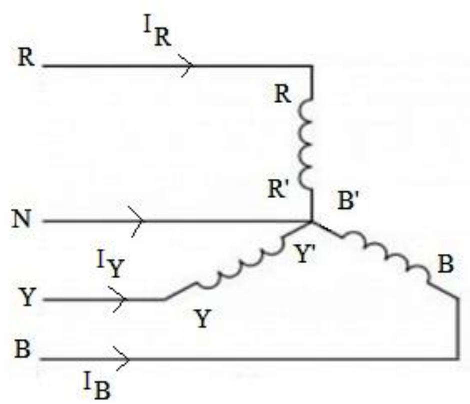


Fig.1.5 Star Connected Syatem

- The voltage across each coil, i.e., the voltage between R & R', Y & Y', and B & B' are called phase voltages(acting from finishing end to starting end).
- $V_{RR'}$, $V_{YY'}$, $V_{BB'}$ or V_{RN}, V_{YN}, V_{BN} represent phase voltages.
- The voltages across line terminals R & Y, Y & B, B & R are called line voltages.
- The connection diagram and the corresponding phasor diagram of voltages is shown in fig.

- From the star connected 3 phase system, it is clearly observed that whatever currents flow through the lines R, Y, B also flow through the respective phase windings.
- Hence in star connected system, the phase currents and line currents are identical.

$$\text{Phase current } (I_{ph}) = \text{Line currents } (I_L)$$

$$I_{ph} = I_{Line}$$

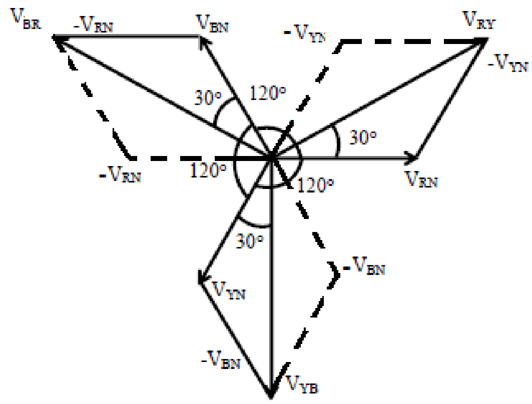


Fig.1.6 Phasor Diagram of Star System

The voltage V_{RY} between lines R and Y is obtained by adding V_{RN} and V_{NY} respectively.

$$V_{RY} = V_{RN}+V_{NY} = V_{RN} - V_{YN}$$

Similarly $$V_{YB} = V_{YN}+V_{NB} = V_{YN} - V_{BN}$$

$$V_{BR} = V_{BN}+V_{NR} = V_{BN} - V_{RN}$$

The line voltage V_{RY} is obtained by adding V_{RN} with reversed vector of V_{YN}.V_{RY} bisects the angle between V_{RN} and $-V_{YN}$

$$V_{RY}^2 = V_L^2 = V_{ph}^2 + V_{ph}^2 + 2\, V_{ph}V_{ph}\cos 60^0 = 3V_{ph}^2$$

$$V_{RY} = \sqrt{3}\, V_{ph}$$

$$\text{Line voltage} = \sqrt{3} \text{ phase voltage}$$

- The line voltages V_{RY}, V_{YB}, V_{BR} are equal in magnitude and differ in phase by 120^0.
- Hence they form a balanced 3-phase voltage of magnitude $\sqrt{3}\, V_{ph}$.
- The two voltages differ in phase by 30^0.
- When the system is balanced, the three phase currents I_R, I_Y, I_B are balanced.

- The magnitude and phase angle of current is determined by circuit parameters.
- I_R, I_Y, I_B are line or phase currents.
- The current in the neutral wire is I_N and is by applying kirchoff's current law at star point, we get

$$I_N = -(I_R + I_Y + I_B)$$

- If the currents are balanced, then the neutral current is zero.

1.2.2 Delta connection or MESH connection:

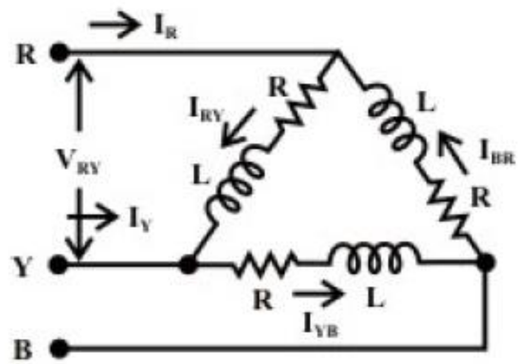


Fig.1.7 Delta Connected System

- The currents flowing through the phase windings $I_{RR'}$, $I_{YY'}$, and $I_{BB'}$ or I_{RY}, I_{YB}, and I_{BR} are called phase currents and are balanced as shown in phasor diagram Fig.1.8.

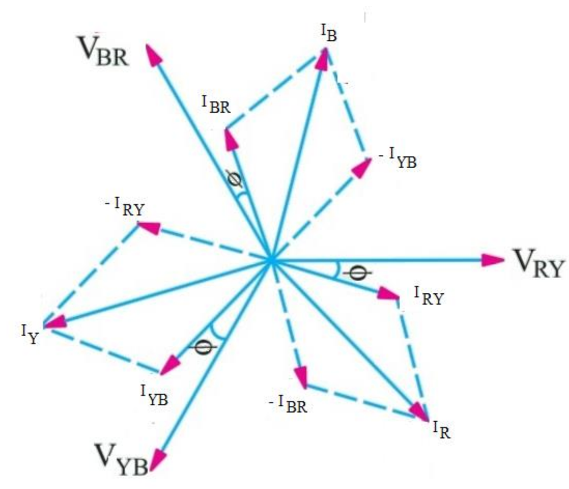


Fig.1.8 Phasor Diagram of Delta System

By applying KCL at node R

$$I_A + I_{BR} = I_{RY},\ I_R = I_{RY} - I_{BR}$$

Similarly by applying KCL at nodes Y and B

$$I_Y = I_{YB} - I_{BR}$$

$$I_B = I_{BR} - I_{YB}$$

The line current I_R is obtained by adding I_{RY} and $-I_{BR}$ vectorially. I_R bisects the angle between I_{RY} and $-I_{BR}$

$$I_R^2 = I_{Line}^2 = I_{ph}^2 + I_{ph}^2 + 2\, I_{ph} I_{ph} \cos 60^0$$

$$= 3I_{ph}^2$$

$$I_L = \sqrt{3}\, I_{ph}$$

- Line current(I_L) = √3 phase voltage(I_{ph})
- The line current I_R, I_Y, I_B and also equal and differ in phase by 120^0. They form a balanced system of currents.
- The line and phase currents differ in phase by 30^0.

1.3 Analysis of balanced three phase circuits

A set of three impedances interconnected in the form of a star or delta form a 3-phase star or delta connected load.

- If the three impedances are identical and equal then it is a balanced 3-phase load, otherwise it is an unbalanced 3-phase load.

The analysis of balanced 3-phase circuits is illustrated as follows

1.3.1 Balanced delta connected load:

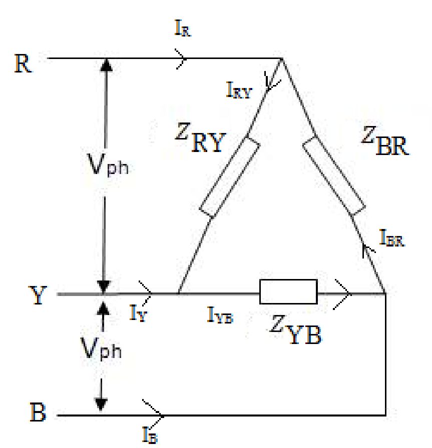


Fig.1.9 Balanced Delta Connected Load

Let us consider a balanced 3-phase delta connected load

Determination of phase voltages:

$$V_{RY} = V\angle 0^0,\ V_{YB} = V\angle\text{-}120^0,\ V_{BR} = V\angle - 240^0 = V\angle 120^0$$

Determination of phase currents:

Phase current = Phase voltage/ Load impedance

$$I_{RY}=\frac{V_{RY}}{Z}\ ;\ I_{YB}=\frac{V_{YB}}{Z}\ ;\ I_{BR}=\frac{V_{BR}}{Z}$$

Determination of line currents:

Line currents are calculated by applying KCL at nodes R,Y,B

$$I_R = I_{RY} - I_{BR}\ ;\ I_Y = I_{YB} - I_{RY}\ ;\ I_B = I_{BR} - I_{YB}$$

- Note: Line currents are also balanced and equal to √3phase current.

1.3.2 Balanced star connected load:

Let us consider a balanced 3-phase star connected load.

For star connection, phase voltage= Line voltage/(√3)

For RYB sequence, the phase voltage is polar form are taken as

$$V_{RN} = V_{ph}\angle -90^0\ ;\ V_{YN} = V_{ph}\angle 150^0\ ;\ V_{BN} = V_{ph}\angle 30^0$$

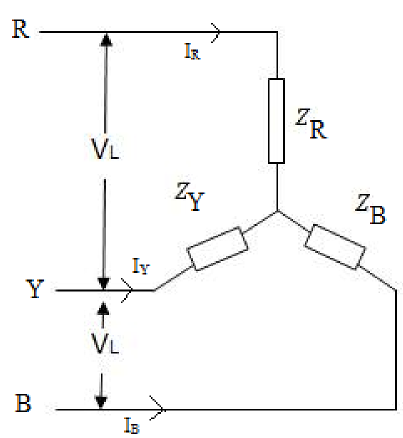


Fig.1.10 Balanced Star Connected Load

For star connection line currents and phase currents are equal

$$I_R = \frac{V_{RN}}{Z}\ ;\ I_Y = \frac{V_{YN}}{Z}\ ;\ I_B = \frac{V_{BN}}{Z}\ ;$$

To determine the current in the neutral wire apply KVL at star point

$$I_N + I_R + I_Y + I_B = 0$$

$$I_N = -(\ I_R + I_Y + I_B)\quad \text{(since they are balanced)}$$

- In a balanced system the neutral current is zero.
- Hence if the load is balanced, the current and voltage will be same whether neutral wire is connected or not.
- Hence for a balanced 3-phase star connected load, whether the supply is 3-phase 3 wire or 3-phase 4 wire, it is immaterial.

- In case of unbalanced load, there will be neutral current.

1.4 Power calculation in three phase balanced system:

- In a balanced 3-phase load, the currents and voltages are balanced.
- Hence the power in each phase is same and hence power calculations are based on per phase basis.
- The total power is given by 3 times the power in each phase.
- If V_{ph} – voltage/ph, I_{ph} – current/phase and the angle between voltage V_{ph} and current I_{ph} is θ then,
  - ✓ Active power/phase = $V_{ph}I_{ph}\cos\theta$ watts/ph
  - ✓ Total active power = 3 $V_{ph}I_{ph}\cos\theta$ watts

Similarly,

- ✓ Reactive power/phase = $V_{ph}I_{ph}\sin\theta$ VAR/ph
- ✓ Total reactive power = 3 $V_{ph}I_{ph}\sin\theta$ VAR
- ✓ Total volt amps = 3 * volt amps/ph =3 $V_{ph}I_{ph}$ volt amps

1.4.1 Expression for power in terms of Line Voltages & Line Currents:

a. Star connected system:

Total power = 3 $V_{ph}I_{ph}\cos\theta$

For star connected systems $V_L = \sqrt{3}\ V_{ph}$;

$$V_{ph} = \frac{V_L}{\sqrt{3}} \text{ and } I_L = I_{ph}$$

- ✓ Total power = $3(\frac{V_L}{\sqrt{3}})I_L\cos\theta$
  $= \sqrt{3}\ V_LI_L\cos\theta$
- ✓ Total reactive power = $\sqrt{3}\ V_LI_L \sin\theta$
- ✓ Total volt amps = $\sqrt{3}\ V_LI_L$

b. Delta connected system:

Total power = 3 $V_{ph}I_{ph}\cos\theta$

For delta connected systems $I_L = \sqrt{3}\ I_{ph}$;

$$I_{ph} = \frac{I_L}{\sqrt{3}} \text{ and } V_L = V_{ph}$$

- ✓ Total power = $3(\frac{I_L}{\sqrt{3}})V_L\cos\theta$
  $= \sqrt{3}\ V_LI_L\cos\theta$
- ✓ Total reactive power = $\sqrt{3}\ V_LI_L \sin\theta$
- ✓ Total volt amps = $\sqrt{3}\ V_LI_L$

- For either balanced star or delta connected systems, the total active power is given by total reactive power = $\sqrt{3}\ V_L I_L \sin\theta$.Where θ angle between phase voltage and phase current.
- The power factor of balanced 3-phase load (either star or delta connected) is the cosine of the angle between phase voltage and phase current.
- In unbalanced circuit, the power, reactive power and apparent power in each phase is different. Hence they have to be calculated separately and to be added to get total power in 3-phase system.

1.5.1 Two watt meter method applied to balanced loads:

In this section, we derive the expression for the readings of watt meters W_1 and W_2 used to measure power in a balanced 3 phase load connected to a balanced 3-phase supply.

Consider a balanced star connected load of impedance $Z\angle\theta$ ohms/ph as shown in Fig1.11.

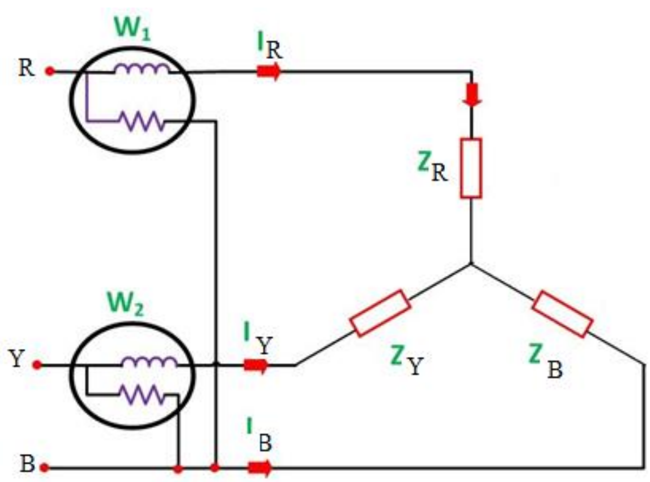


Fig.1.11 Two Wattmeter method to measure Active Power

The phasor diagram of voltage and currents are shown in Fig.1.12

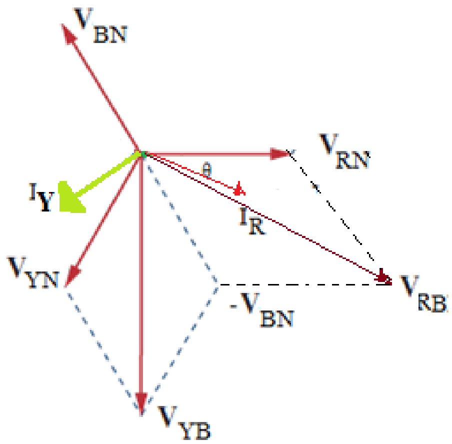


Fig.1.12

The phasor diagram is drawn for the RYB sequence is as shown.

Watt meter W_1:

- Current through current coil = I_R
- Voltage across pressure coil = $V_{RB} = V_{RS} - V_{BS}$
- Phase difference between V_{RB} and I_R = $30 - \theta$
- Power measured by $W_1 = V_{RB} I_R \cos(30 - \theta)$
- Since the load is balanced, and the supply is also balanced V_{RB} and I_R represent line voltage and line currents respectively.
- Reading of $W_1 = V_L I_L \cos(30 - \theta)$

Watt meter W_2:

- Current through current coil = I_Y
- Voltage across pressure coil = $V_{YB} = V_{YS} - V_{BS}$
- Phase difference between V_{YB} and I_Y = $30 + \theta$
- Power measured by $W_2 = V_{YB} I_Y \cos(30+\theta)$
  $= V_L I_L \cos(30+\theta)$ (1)
- The total power is given by algebraic sum of the watt meter readings.
- $W_1 + W_2 = V_L I_L [\cos(30 - \theta) + \cos(30+\theta)]$
  $= V_L I_L \, 2 \cos 30^0 \cos\theta = \sqrt{3} \, V_L I_L \cos\theta$
- $W_1 + W_2 = \sqrt{3} \, V_L I_L \cos\theta$ = total power (2)
- The difference in the wattmeter reading:
  $W_1 - W_2 = V_L I_L [\cos(30 - \theta) - \cos(30+\theta)]$
  $= V_L I_L \, 2 \sin 30^0 \sin\theta = V_L I_L \sin\theta$
- Total reactive power = $\sqrt{3}(W_1 - W_2) = \sqrt{3} \, V_L I_L \sin\theta$ (3)
- Dividing (3) by (2) we get,
  $\tan\theta = \sqrt{3}(W_1 - W_2)/W_1 + W_2$ (4)

From the above equation (4) we can calculate phase angle θ and hence power factor cos θ can be determined from the watt meter readings. θ is considered +ve for lagging p.f and –ve for leading p.f.

1.5.2. Measurement of Reactive Power:

Reactive Power can be measured only for balanced Loads.
To measure Reactive Power Wattmeter current coil will be placed in one of the line and Pressure coil between the remaining two line terminals as shown in figure1.13

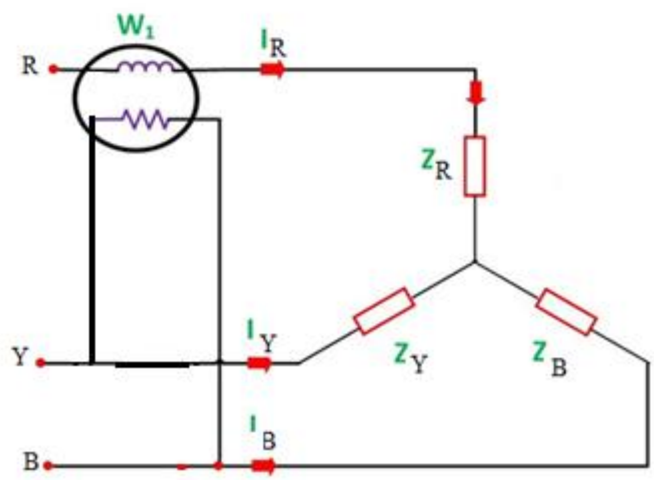


Fig.1.13 Single Wattmeter method to measure Reactive Power

Power measured by the wattmeter W_1 can be obtained from the product of current through current coil(I_R) and voltage measured by the pressure coil(V_{YB})

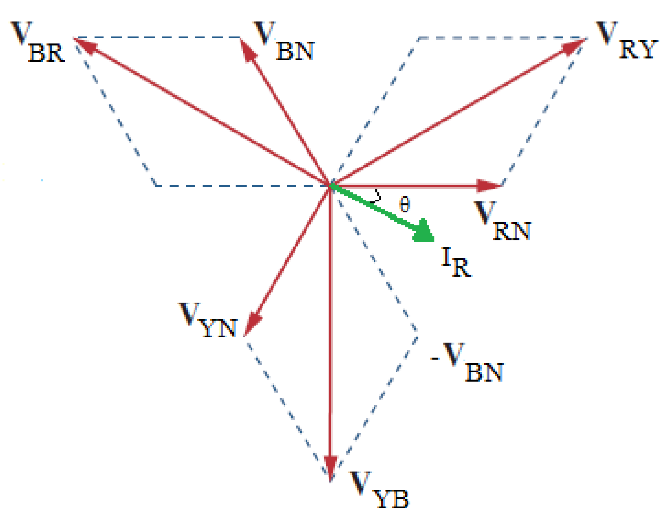


Fig.1.14 Phasor Diagram

From the phasor diagram angle between V_{YB} and I_R is $90-\theta$
Power measured by wattmeter W_1 is given by $W_1 = V_{YB} I_R \cos(90-\theta)$

$$= V_{YB} I_R \sin\theta$$
$$= V_L I_L sin\ \theta$$

Reactive Power $=\sqrt{3}\, V_L I_L sin\ \theta = \sqrt{3}\, W_1$
By using this method one can measure reactive Power as $\sqrt{3}$ times of wattmeter reading.

UNIT - III

Unbalanced Three Phase Circuits

Objectives:

- To analyze three phase Unbalanced systems
- To Measure active and reactive power in Three Phase Unbalanced systems.

Syllabus:

Analysis of three phase Unbalanced circuits- Loop method-Application of Millman's theorem-Star Delta transformation Technique-Measurement of power

Outcomes:

On completion the student should be able to:

- Describe the reasons for, and the generation of the Unbalanced Voltages and Circulating currents.
- Solve three-phase circuits in terms of phase and line quantities, and the power developed in three-phase Unbalanced loads.
- Measure power dissipation in Unbalanced three-phase loads.

3.1 Introduction:

An unbalanced three-phase circuit is one that contains at least one source or load that does not possess three-phase symmetry. A source with the three source-function magnitudes unequal and/or the successive phase displacements different from 120° can make a circuit unbalanced. Similarly, a three-phase load with unequal phase impedance values can make a circuit unbalanced.

The single-phase equivalent circuit technique of analysis *does not* work for unbalanced three-phase circuits. General circuit analysis techniques like mesh analysis or nodal analysis will have to be employed for analyzing such circuits.

3.2 Analysis of Three phase unbalanced circuits:

3.2.1 Unbalanced delta connected load

Let us consider an unbalanced delta connected load fed from a 3-phase 3 wire balanced supply. Since the terminals are fixed, the voltage drop across each load impedance is known. Hence the current in each load impedance can be computed and then apply KCL at junctions to obtain the line currents.

- The method of solution is similar to that of a balanced delta connected load.
- But the phase currents will neither be equal in magnitude nor have a phase difference of 120^0.

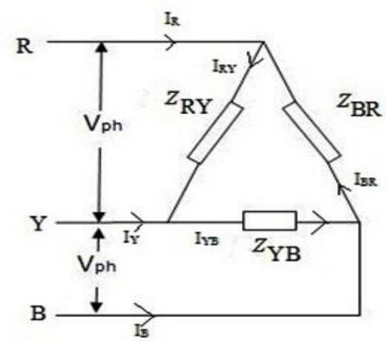


Fig 3.1 Unbalanced Delta Load

Determination of phase voltages:

$$V_{AB} = V\angle 0^0,\ V_{BC} = V\angle -120^0,\ V_{CA} = V\angle -240^0 = V\angle 120^0$$

Phase currents are computed as

$$I_{AB} = \frac{V_{AB}}{Z_{AB}}\ ;\ I_{BC} = \frac{V_{BC}}{Z_{BC}}\ ;\ I_{CA} = \frac{V_{CA}}{Z_{CA}}$$

Determination of Line currents:

By applying KCL at junction R, Y, B we get,

$$I_A = I_{AB} - I_{BC};\ I_B = I_{BC} - I_{CA};\ I_C = I_{CA} - I_{AB};$$

$$\text{Check: } I_A + I_B + I_C = 0$$

- The sum of the line currents in a 3-phase 3-wire system is zero.

3.2.2 Unbalanced star connected load with neutral

Let us consider an unbalanced star connected load which is fed from a 3-phase 4 wire supply. The neutral of the supply is connected to the star point of the load i.e., the star point of the load and neutral are at the same potential (ground potential). The voltage across each of the load impedance is known and is equal to the line to neutral voltage (phase voltage). The currents in each of the load impedances can be computed and they will be line currents and they are unbalanced.

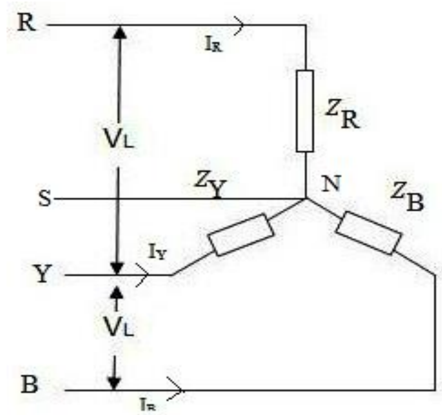


Fig.3.2 Unbalanced Star Load with Neutral

- Hence in an unbalanced system the neutral wire will carry current and forms the return path for the phase currents.
- The analysis of 3-phase 4-wire star connected unbalanced load is simple compared to 3-phase 3 wire star or delta connected loads.

Determination of phase voltages

$$V_{RS}= \frac{V}{\sqrt{3}}\angle 0^0 \text{ ; } V_{YS} = \frac{V}{\sqrt{3}}\angle -120^0 \text{ ; } V_{BS} = \frac{V}{\sqrt{3}}\angle 120^0 \text{ ;}$$

Determination of line currents

$$I_R= \frac{V_{RS}}{Z_R} \text{ ; } I_Y = \frac{V_{YS}}{Z_Y} \text{ ; } I_B = \frac{V_{BS}}{Z_B} \text{ ;}$$

$$\text{Neutral current } I_N = -(I_R + I_Y + I_B)$$

3.2.3 Unbalanced star connected load without neutral:

An unbalanced star connected load is supplied from a balanced 3-phase 3 wire supply. Since the load is unbalanced, the voltages across each load impedance are not equal to phase voltage but it is different. The voltages across each load impedance if it is determined, then we can determine the line currents.

- Since the voltage across Z_A, Z_B, Z_C are different the voltage of the star point S of the 3-phase load, and of the neutral point of the supply are different.
- The potential difference between the neutral point of the supply N and star point S of the load is called Neutral displacement or Neutral shift.

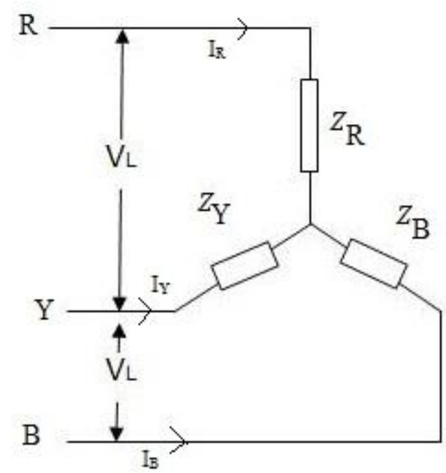


Fig.3.3 Unbalanced star connected load without neutral

3.3 Loop Method:

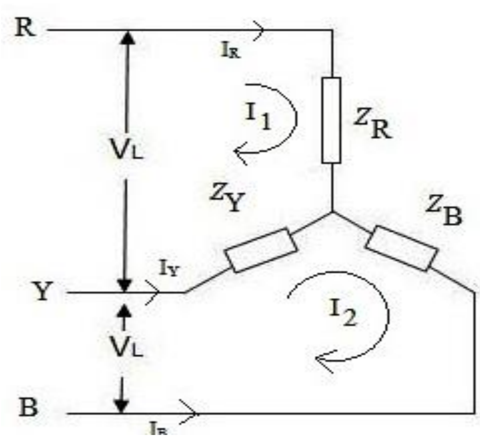


Fig.3.4 Unbalanced Star Load for Loop Analysis

The voltage V_{AN} is taken as reference. Applying KVL for each of the loops.

Loop 1:

$$I_1Z_R + (I_1 - I_2)Z_Y = V_{RY}$$

$$I_1(Z_R + Z_Y) - I_2(Z_Y) = V_{RY}$$

Loop 2:

$$(I_2 - I_1)\ Z_Y + I_2Z_B = V_{YB}$$

$$-I_1(Z_Y) + I_2(Z_Y + Z_B) = V_{YB}$$

Writing down the above equations in matrix form

$$\begin{bmatrix} Z_R+Z_Y & -Z_Y \\ -Z_Y & Z_Y+Z_B \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \begin{bmatrix} V_{RY} \\ V_{YB} \end{bmatrix}$$

By using cramer's rule we will get I_1 and I_2

The line currents I_R, I_Y and I_B are given by

$$I_R = I_1 \; ; \; I_Y = I_2 - I_1 \; ; \; I_B = -(I_2)$$

The voltage across each load impedance

$$V_{RN} = I_R * Z_R; \; V_{YS} = I_Y * Z_Y \; ; \; V_{BS} = I_B * Z_B \; ;$$

The neutral displacement voltage V_{NS}

$$V_{NS} = V_{RN} - V_{RS}$$

3.4 Milliman's Theorem:

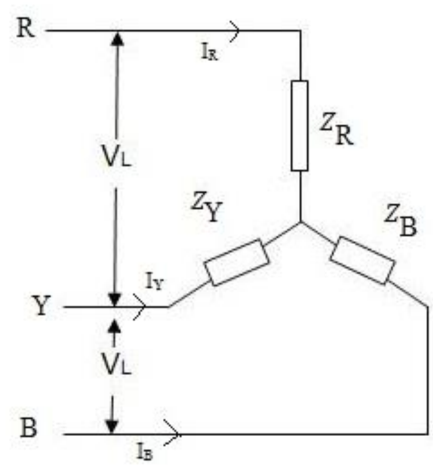


Fig.3.5 Star Connected Load

In this method, the neutral shift voltage (V_{NS}) is determined by using the following expression derived below:

$$V_{NS} = -\frac{(V_{RN}Y_R + V_{YN}Y_Y + V_{BN}Y_B)}{Y_R + Y_Y + Y_B}$$

- The line to neutral voltages V_{RN} , V_{YN} , and V_{BN} are the balanced phase voltages obtained from the supply.
- Y_R , Y_Y and Y_B are the star connected load admittances.

The voltages across the load impedances

$$V_{RS} = V_{RN} + V_{NS}$$
$$V_{YS} = V_{YN} + V_{NS}$$
$$V_{BS} = V_{BN} + V_{NS}$$

The line currents are given by

$$I_R = \frac{V_{RS}}{Z_R} \; ; \; I_Y = \frac{V_{YS}}{Z_Y} \; ; \; I_B = \frac{V_{BS}}{Z_B} \; ;$$

Applying KCL at the star point S,

$$I_R + I_Y + I_B = 0$$

$$\frac{V_{RS}}{Z_R} + \frac{V_{YS}}{Z_Y} + \frac{V_{BS}}{Z_B} = 0$$

3.5 Star / Delta Conversion:

For solving an unbalanced star connected load, we will replaced it by an equivalent delta connected load and solve the delta connected load. The principle to get an equivalent delta connected load is to equate the impedances between corresponding terminals of the two loads as shown below.

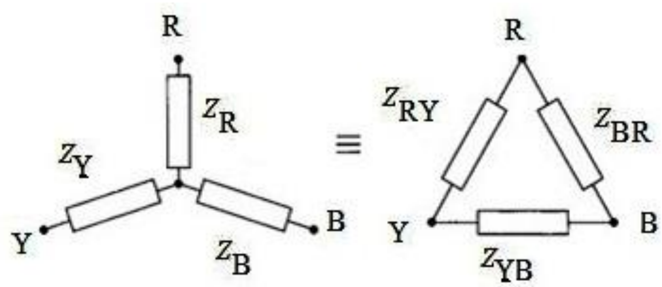


Fig.3.6 Star Delta Equivalents

For delta to star conversion

$$Z_R = \frac{Z_{RY}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

$$Z_Y = \frac{Z_{RY}Z_{YB}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

$$Z_B = \frac{Z_{YB}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

$$Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}$$

$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}$$

$$Z_{BR} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}$$

we can replace a star connected load by an equivalent delta connected load ($\lambda - \Delta$ conversion)

Phase currents are computed as

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} \; ; \; I_{YB} = \frac{V_{YB}}{Z_{YB}} \; ; \; I_{BR} = \frac{V_{BR}}{Z_{BR}}$$

Determination of Line currents:

By applying KCL at junction R, Y, B we get,

$$I_R = I_{RY} - I_{BR}; \; I_Y = I_{YB} - I_{RY} \; ; \; I_B = I_{BR} - I_{YB} \; ;$$

Check: $I_R + I_Y + I_B = 0$

3.6 Measurement of Power:

3.6.1 Two wattmeter method:

- This is the most common method of measurement of power in 3-phase circuits.
- This method can be employed for balanced or unbalanced, star or delta connected 3-phase circuits.

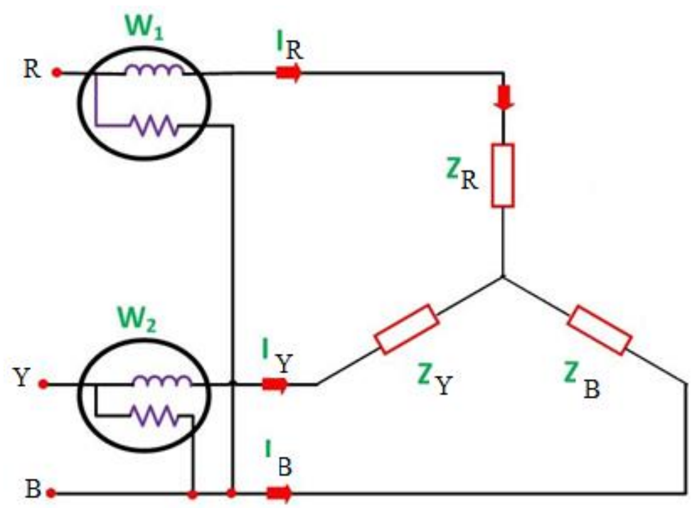


Fig.1.17 Two Wattmeter method to measure Active Power in unbalanced Load

Let us consider a 3-phase star connected load of impedances Z_R , Z_Y, and Z_B and the two watt meters are connected to measure total power.

The current coils of the two watt meters W_1 and W_2 are connected in two lines R and Y, the potential coils W_1 and W_2 are connected between lines R-B and Y-B.

Let V_{RS}, V_{YS} and V_{BS} be the instantaneous values of voltage drops across the load impedances.

Let i_R, i_Y and i_B be the instantaneous values of currents in the load impedances.

The total instantaneous power in the 3-phase load

$$= V_{RS}\, i_R + V_{YS} i_Y + V_{BS} i_B \tag{1.1}$$

From Fig., we see that instantaneous current through W_1 is i_R and the instantaneous voltage across the pressure coil of W_1 is V_{RB} and hence the instantaneous power measured by W_1 is

$$W_1 = i_R V_{RB} = i_R[V_{RS} - V_{BS}] \tag{1.2}$$

Similarly the instantaneous power measured by W_2 is

$$W_2 = i_Y V_{YB} = i_Y[V_{YS} - V_{BS}] \tag{1.3}$$

Sum of the instantaneous power read by W_1 and W_2 is

$$W_1 + W_2 = i_R[V_{RS} - V_{BS}] + i_Y[V_{YS} - V_{BS}]$$

$$= i_R V_{RS} + i_Y V_{YS} - V_{BS}[i_R + i_Y] \tag{1.4}$$

Applying KCL to node s, i.e., star point, we get,

$$i_R + i_Y + i_B = 0$$

$$i_R + i_Y = -\, i_B \tag{1.5}$$

substituting equation (1.5) in equation (1.4) we get,

$$W_1 + W_2 = i_R V_{RS} + i_Y V_{YS} + i_B V_{BS} \tag{1.6}$$

- Since equation (1.1) and (1.6) are identical, the sum of the two watt meter readings given the total instantaneous power.
- Actually the power measured by each watt meter varies from instant to instant. But inertia of the moving systems makes the pointer to read the average power.

- The above proof does not assume a balanced load or a sinusoidal wave form hence is applicable under all conditions.

Assignment-Cum-Tutorial Questions

SECTION-A

1. An unbalanced system is caused by

a) The source voltages are not equal in magnitude

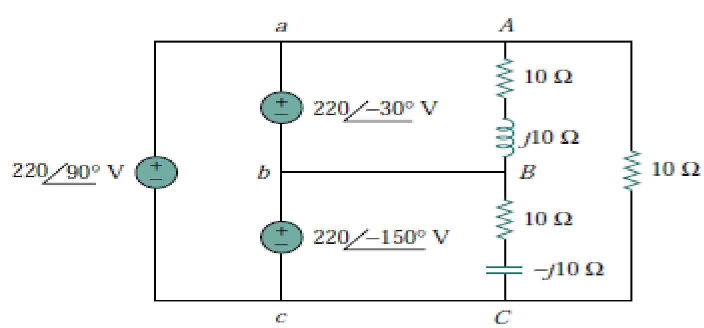
b) Difference in phase by angles that are unequal

c) Load impedances are unequal.

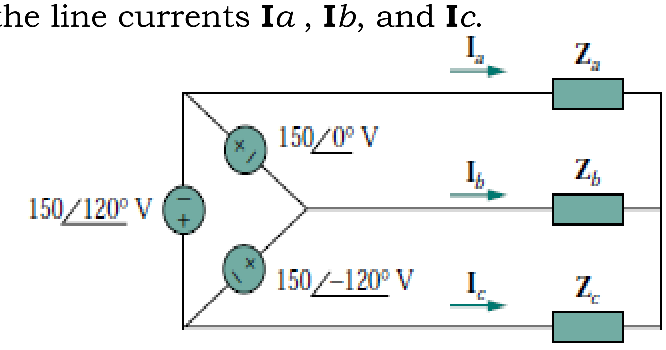
d) All the above

2. A 400 V, 3-phase, 4 wire, star-connected system supplies three resistive loads of 15 kW, 20 kW and 25 kW in the red, yellow and blue phases respectively. Determine the current flowing in each of the four conductors.

3. For the unbalanced circuit in Figure below, Find the generator current $\mathbf{I}_{ca}$, the line current $\mathbf{I}_{cC}$, and the phase current $\mathbf{I}_{AB}$.



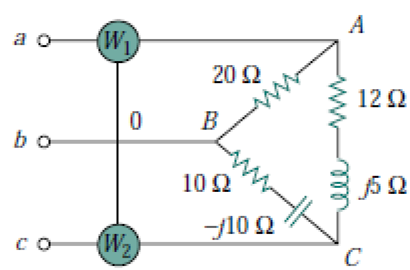
4. For the circuit in Figure shown below, $\mathbf{Z}_a = 6 - j8$, $\mathbf{Z}_b = 12 + j9$, and $\mathbf{Z}_c = 15$. Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.



5. A delta-connected load whose phase impedances are $\mathbf{Z}AB = 50$, $\mathbf{Z}BC = -j50$, and $\mathbf{Z}CA = j50$ is fed by a balanced wye-connected three-phase source with $Vp = 100$ V. Find the phase currents.

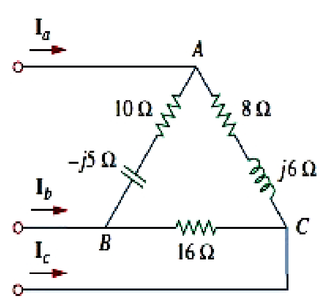
6. A balanced three-phase wye-connected generator with $Vp = 220$ V supplies an unbalanced wye-connected load with $\mathbf{Z}AN = 60 + j80$, $\mathbf{Z}BN = 100 - j120$, and $\mathbf{Z}CN = 30 + j40$. Find the total complex power absorbed by the load.

7. In Figure, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $\mathbf{V}ab = 208$ V with positive phase sequence.
   (a) Determine the reading of each wattmeter. (b) Calculate the total apparent power absorbed by the load.

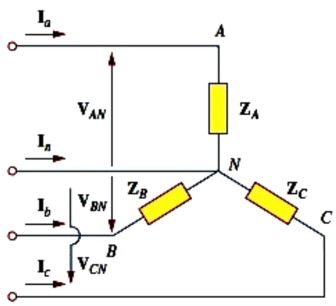


SECTION-B

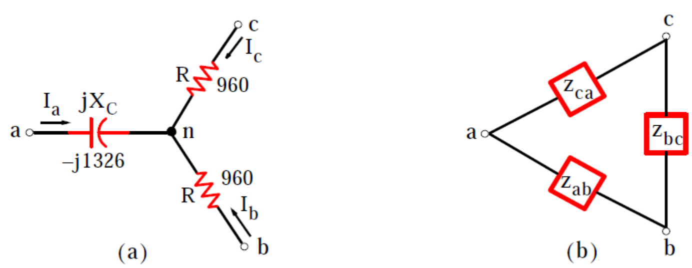
1. The unbalanced Δ load of Fig. is supplied by balanced line-to-line voltages of 440 V in the positive sequence. Find the line currents. Take V_{ab} as reference.



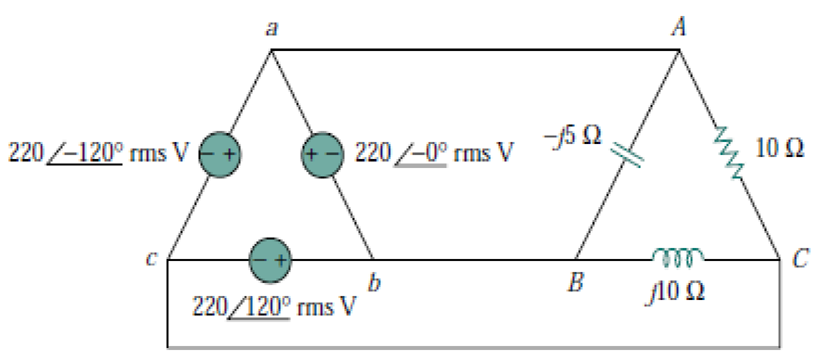
2. The unbalanced Y-load of Fig has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take Z_A=15Ω, Z_B=(10+j5)Ω, Z_C=(6-j8)Ω



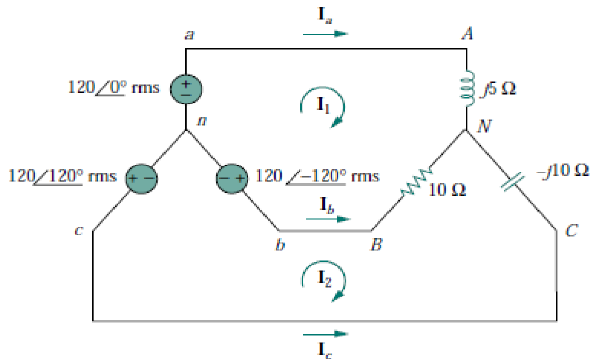
3. For the phase sequence indicator as shown in Figure find the equivalent



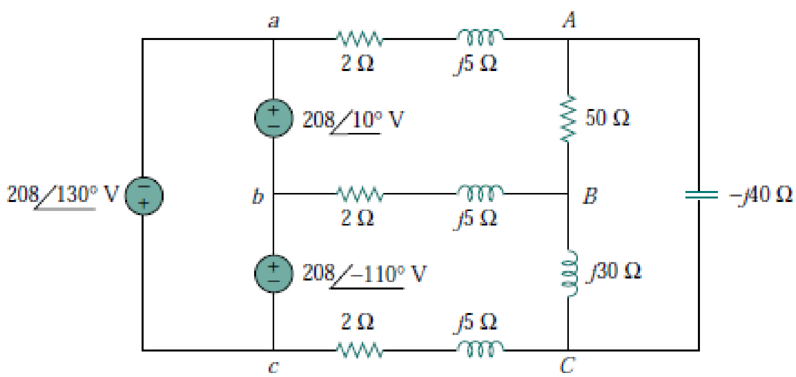
4. Find the line currents in the unbalanced three-phase circuit of Figure and the real power absorbed by the load.



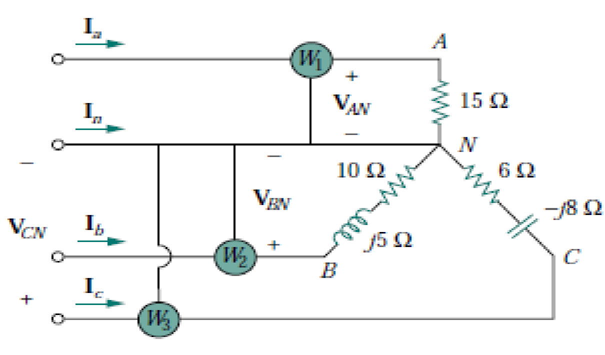
5. For the unbalanced circuit in Figure find:
   (a) the line currents,
   (b) the total complex power absorbed by the load, and
   (c) the total complex power supplied by the source.



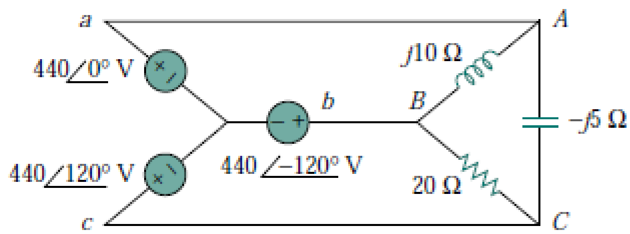
6. Consider the unbalanced *circuit* shown in Figure below. Find the generator current $\mathbf{I}_{ab}$, the line current $\mathbf{I}_{bB}$, and the phase current $\mathbf{I}_{BC}$.



7. Three watt meters $W1$, $W2$, and $W3$ are connected, respectively, to phases a, b, and c to measure the total power absorbed by the unbalanced wye connected load.
   (a) Predict the wattmeter readings. (b) Find the total power absorbed.



8. Refer to the unbalanced circuit of Figure.
   Calculate:
   (a) the line currents
   (b) the real power absorbed by the load
   (c) the total complex power supplied by the source



SECTION-C

1. A good phase sequence indicator operates with one lamp very bright and the other very dim. Using the same lamps as in figure but with a capacitor of different value, can you design a better indicator?

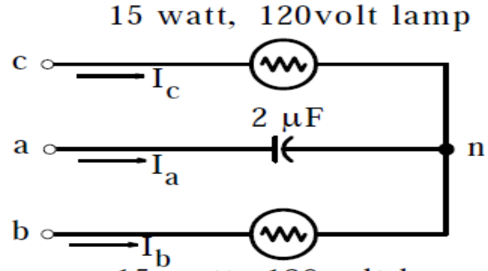
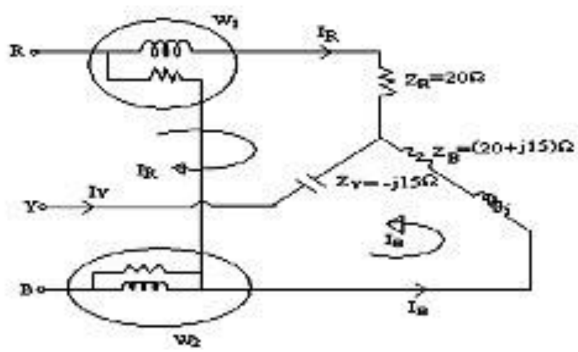
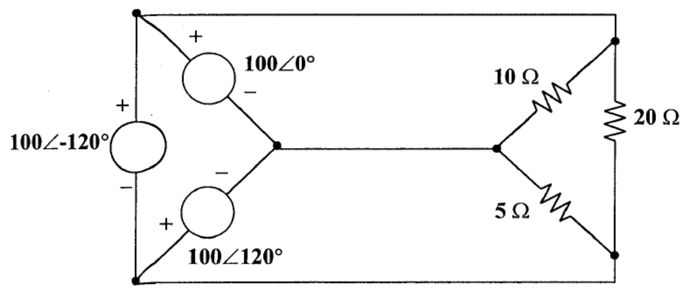


Figure shows a typical phase indicator consisting of two resistors representing two light bulbs each rated 15 watts, 120 volts at 60Hz frequency, and a capacitor connected to a120 volt three phase system.

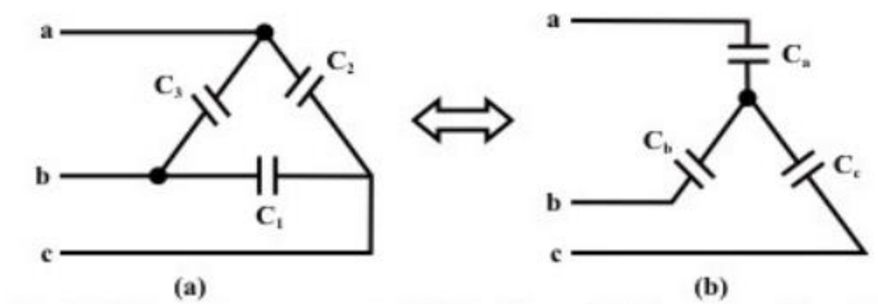
2. An unbalanced star connected load is connected across a 3-φ, 400V balanced supply of phase sequence RYB as shown in fig. Two wattmeters are connected to measure the total power supplied as shown in fig. Find the readings of the wattmeters.



3. Given an unbalanced delta connected load, obtain the respective phase currents.



4. Find out the equivalent capacitance when the following transformation is used



5. A 230V (phase), 50 Hz, three phase, 4-wire system has a phase sequence ABC. A unity power factor load of 4 kW is connected between phase A and neutral N. It is desired to achieve zero neutral current through the use of a pure inductor and a pure capacitor in the other two phases. The values of inductor and capacitor are

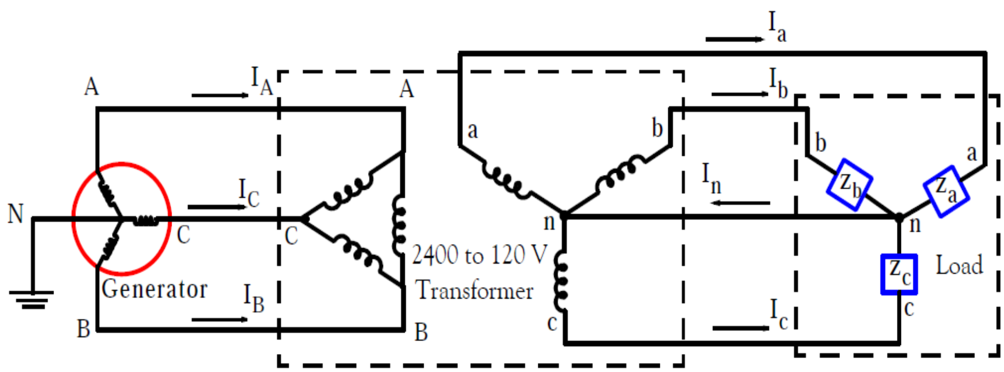
(a) 72.95 mH in phase C and 139.02 μF in phase B.

(b) 72.95 mH in phase B and 139.02 μF in phase C.

(c) 42.12 mH in phase C and 240.79 μF in phase B.

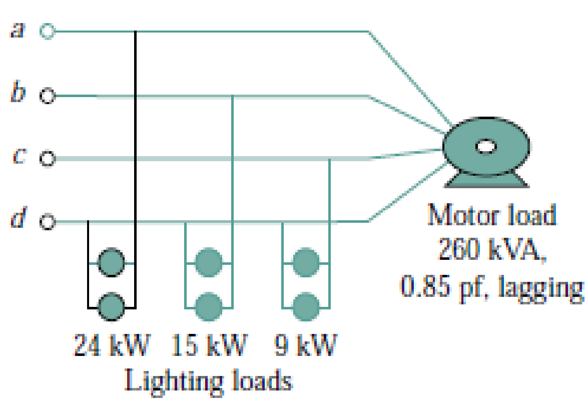
(d) 42.12 mH in phase B and 240.79 μF in phase C.

6. For the three phase system in Figure, compute the generator voltages Vab, Vbc , and Vca . Assume that each transformer impedance on the high side is j30 and the transformer resistances are negligible. Assume also that the lines are very short and thus their impedances can are also negligible.



7. As shown in Figure a three-phase four-wire line with a phase voltage of 120 V supplies a balanced motor load at 260 kVA at 0.85 pf lagging. The motor load is connected to the three main lines marked *a*, *b*, and *c*. In addition, incandescent lamps (unity pf) are connected as follows: 24 kW from line *a* to the neutral, 15 kW from line *b* to the neutral and 9 kW from line *a* to the neutral.

(a) If three watt meters are arranged to measure the power in each line, calculate the reading of each meter.
(b) Find the current in the neutral line.



UNIT - IV

Transient Analysis for D.C Excitation

Objectives:

- To introduce the concept of Transients in electrical circuits.
- To study the transient behavior of RL, RC & RLC circuits for DC excitation.

Syllabus:

Transient response of series R-L, R-C and R-L-C circuits for DC excitation-Initial Conditions-Solution method using differential equation approach and Laplace transform method

Outcomes:

On completion the student should be able to:

- Calculate the time constant for RL and RC circuits.
- Analyze the transient behavior of first order and second order circuits for DC excitations using differential equation approach & Laplace transform approach.
- Obtain the transformed networks and find the response using inverse Laplace transforms.

4.1 Introduction to Transients

Transient analysis (or just transients) of electrical circuits is as important as steady-state analysis. When transients occur, the currents and voltages in some parts of the circuit may many times exceed those that exist in normal behavior and may destroy the circuit equipment in its proper operation. We may distinguish the transient behavior of an electrical circuit from its steady-state, in that during the transients all the quantities, such as currents, voltages, power and energy, are changed in time, while in steady-state they remain invariant, i.e. constant (in d.c operation) or periodical (in a.c operation) having constant amplitudes and phase angles.

The cause of transients

- change in circuit parameters and/or in circuit configurations, which usually occurs as a result of switching (commutation),
- short, and/or open circuiting,
- Change in the operation of sources etc.

The transient processes are attained by the interchange of energy, which is usually stored in the magnetic field of inductances or/and the electrical field of capacitances. Any change in energy cannot be abrupt otherwise it will result in infinite power (as the power is a derivative of energy, p=dw/dt), which is in contrast to physical reality. All transient changes, which are also called transient responses (or just responses), vanish and, after their disappearance, a new steady-state operation is established. In this respect, we may say that the transient describes the circuit behavior between two steady states: an old one, which was prior to changes, and a new one, which arises after the changes.

A few methods of transient analysis are known: the classical method, The Cauchy-Heaviside (C-H) operational method, the Fourier transformation method and the Laplace transformation method.

Comparing the classical method and the laplace transformation method it should be noted that the latter requires more knowledge of mathematics and is less related to the physical matter of transient behavior of electric circuits than the former.

Classical method of transient analysis is based on the determination of differential equations and splitting the solution into two components: natural and forced responses. The classical method is fairly complicated mathematically, but is simple in engineering practice.

4.2 Natural and Forced Responses:

Solving differential equations by the classical method, complete solution of any linear differential equation as composed of two parts: the complementary solution (or natural response) and the particular solution (or forced response). To understand these principles, let us consider a first order differential equation,

$$\frac{dv}{dt} + P(t)v = Q(t) \qquad (4.1)$$

Here Q(t) is identified as a forcing function, which is generally a function of time (or constant, if a d.c. source is applied) and P(t), is also generally a function of time, represents the circuit parameters. In our study, however, it will be a constant quantity, since the value of circuit elements does not change during the transients (indeed, the circuit parameters do change during the transients, but we may neglect this change as in many cases it is not significant).

$$v = e^{-Pt} \int Qe^{Pt} dt + Ae^{-Pt} \qquad (4.2)$$

General solution can be written as

$$v = v_{PI} + v_{CF}$$

$$v_{PI} = \text{Particular integral} = e^{-Pt} \int Qe^{Pt} dt$$

$$v_{CF} = \text{Complementary Function} = Ae^{-Pt}$$

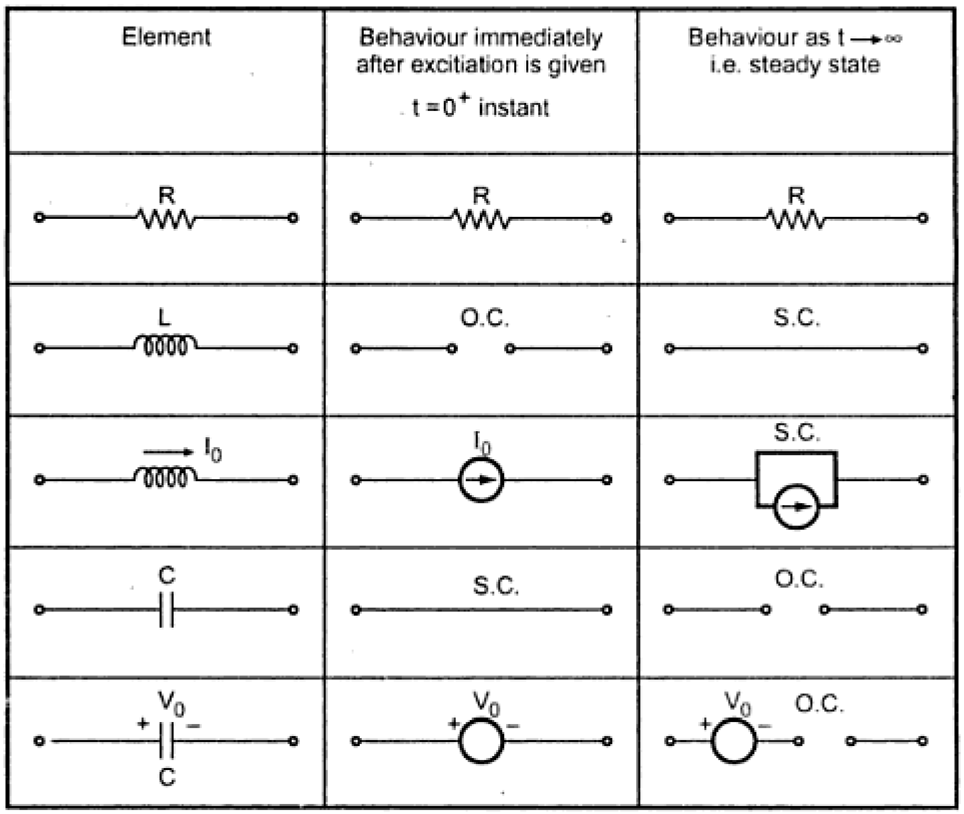
In general v_{PI} may be written as a Steady state value, designated asV_{ss}. Remaining part V is called Transient portion of solution V_t.

$$V = V_{ss} + V_t$$

$$V_{PI} = V_{ss} = \text{Source response} = \text{Steady state response} = \text{Forced response}$$

$$V_{CF} = V_t = \text{Source free response} = \text{Transient response} = \text{Natural response}$$

Complete solution is composed of two parts. The first one, which is dependent on the forcing function Q, is the forced response (it is also called the steady state response or the particular solution or the particular integral). The second one, which does not depend on the forcing function, but only on the circuit parameters P (the types of elements, their values, interconnections, etc) and on the initial conditions A, i.e., on the "nature" of the circuit, is the natural response. It is also called the solution of the homogeneous equation, which does not include the source function and has anything but zero on its right side. Following this rule, we will solve differential equations by finding natural and forced responses separately and combining them for a complete solution.



4.3 Transient response of series RL, RC, RLC circuits for DC excitation

4.3.1 Transient Response of series RL circuit for DC excitation

Consider series RL circuit shown in Fig.2.1. Consider that switch closed at t=0 and before that switch is open for a long time.

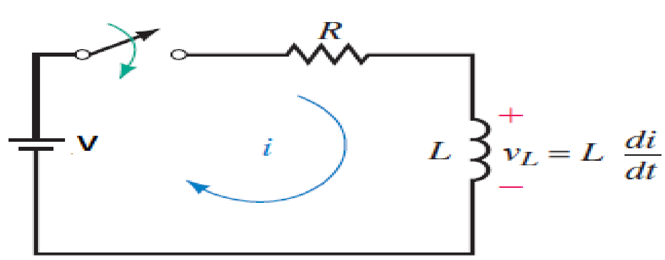
Initial condition

At $t = 0^-$, Switch is open

$$i(0^-) = I_0 = 0$$

And also we know that inductor does not allow sudden change in current through it.

i.e $i(0^+) = i(0^-) = I_0 = 0$



For $t \geq 0$ the switch is closed

Apply KVL to the circuit

$\mathrm{Ri(t)} + \mathrm{L}\frac{\mathrm{di(t)}}{\mathrm{dt}} = \mathrm{V}$

Rearranging

$\frac{\mathrm{di(t)}}{\mathrm{dt}} + \frac{\mathrm{R}}{\mathrm{L}}\mathrm{i(t)} = \frac{\mathrm{V}}{\mathrm{L}}$

The solution to above equation is $i(t) = \frac{\mathrm{V}}{\mathrm{R}} + ce^{-\frac{\mathrm{R}}{\mathrm{L}}t}$ (4.3)

To find c

To find the arbitrary constant 'c' use the initial condition $i(0^+) = i(0^-) = I_0 = 0$

$$\mathrm{i(0)} = 0 = \frac{\mathrm{V}}{\mathrm{R}} + \mathrm{c}$$

$$\mathrm{c} = -\frac{\mathrm{V}}{\mathrm{R}}$$

Substituting c in equation 4.3 we get

$$\boxed{\mathrm{i(t)} = \frac{\mathrm{V}}{\mathrm{R}} - \frac{\mathrm{V}}{\mathrm{R}}\mathrm{e}^{-\frac{\mathrm{R}}{\mathrm{L}}\mathrm{t}}}$$

$$\boxed{\mathrm{i(t)} = \frac{\mathrm{V}}{\mathrm{R}}(1 - \mathrm{e}^{-\frac{\mathrm{R}}{\mathrm{L}}\mathrm{t}})}$$

The point P shown in Fig.2.2 denotes that current rise in the circuit rises to 0.632 times maximum value in steady state. The time required for current to rise to 0.632 of its final value is known as time constant of given RL circuit. It is denoted by τ

$$\boxed{\tau = \frac{\mathrm{L}}{\mathrm{R}}\ \mathrm{sec}}$$

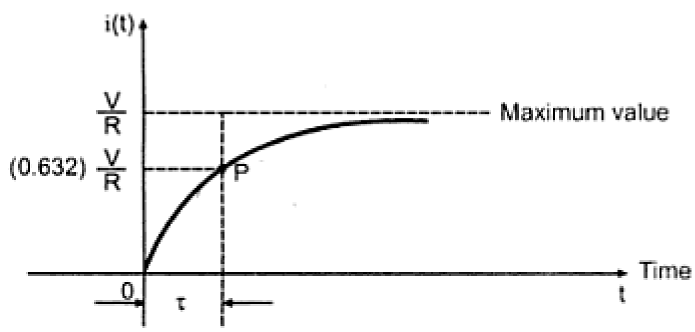


Fig. 2.2

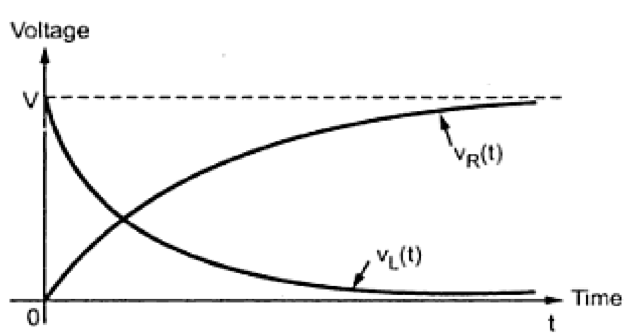


Fig. 2.3 Variation of $v_R(t)$ and $v_L(t)$ with time

Significance of Time constant

To study significance of Time constant substitute different values of t in i(t)

At $t = \tau$, $\quad i(t) = \frac{V}{R}(1 - e^{-1}) = 0.632\frac{V}{R}$

At $t = 2\tau$, $\quad i(t) = \frac{V}{R}(1 - e^{-2}) = 0.8646\frac{V}{R}$

At $t = 4\tau$, $\quad i(t) = \frac{V}{R}(1 - e^{-4}) = 0.9816\frac{V}{R}$

At $t = 6\tau$, $\quad i(t) = \frac{V}{R}(1 - e^{-6}) = 0.9975\frac{V}{R}$

From above values up to first time constant period, the initial rate of raise in current is high. But after one time constant period, this rate slows down for further period of time. Ideally the current reaches steady state value at infinite time, but practically the current reaches steady state value after $t = 6\tau$ or 8τ.

The voltage across the Inductor

$$V_L = L\frac{di(t)}{dt}$$

$$V_L = L\frac{d}{dt}\left[\frac{V}{R}(1 - e^{-\frac{R}{L}t})\right]$$

$$V_L = L\left[0 - \left(\frac{V}{R}\right)\left(-\frac{R}{L}\right)e^{-\frac{R}{L}t})\right]$$

$$\boxed{V_L(t) = Ve^{-\frac{R}{L}t}}$$

The voltage across the Resistance

$$V_R = Ri(t)$$

$$\boxed{V_R(t) = V(1 - e^{-\frac{R}{L}t})}$$

4.3.2 Transient Response of series RC circuit for DC excitation

Consider series RC circuit shown in Fig.2.4. Consider that switch is closed at t=0 and before that switch is open for a long time.

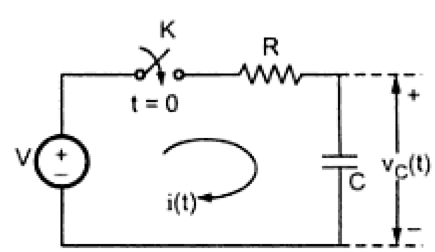


Fig. 2.4

To find transient response of driven series RC circuit means to find expression for voltage across capacitor $V_C(t)$

Initial condition

At $t = 0^-$, Switch is open

Before closing the switch active source is not presented in the circuit, so the initial voltage across capacitor is zero.

$$V_C(0^-) = V_0 = 0$$

And also we know that Capacitor does not allow sudden change in voltage across it.

i.e $V_C(0^+) = V_C(0^-) = V_0 = 0$

For t $\geq$ 0 the switch is closed

Now the voltage source is introduced in the circuit.

Apply KVL to the circuit

$$Ri(t) + \frac{1}{C}\int i(t)dt = V \qquad (4.4)$$

Here i(t) is the current in the circuit also flows through the capacitor.

Differentiating the equation w.r.t time

$$R\frac{di(t)}{dt} + \frac{1}{C}i(t) = 0 \qquad (4.5)$$

$$i(t) = Ke^{-t/RC} \qquad (4.6)$$

K= Arbitrary Constant

To find K

To find K use the initial condition at $t = 0$, $V_C(0^+) = V_C(0^-) = V_0 = 0$

Substitute t=0 $V_C(t) = 0$ in equation 2.4

$$Ri(0) = V \Rightarrow i(0) = \frac{V}{R}$$

Substituting t=0 in equation 4.6

$$\frac{V}{R} = K$$

Substitute K value in equation 4.6

$$\boxed{i(t) = \frac{V}{R}e^{-t/RC}}$$

Above expression is combination of steady state response 0 and transient response

$$\frac{V}{R}e^{-t/RC}$$

In the above equation RC is time constant of the series RC circuit.

$$\tau = RC \text{ sec}$$

Significance of Time constant

To study significance of Time constant substitute different values of t in i(t)

At $t = \tau$, $\quad i(t) = \frac{V}{R}e^{-1} = 0.3679\ \frac{V}{R}$

At $t = 2\tau$, $\quad i(t) = \frac{V}{R}e^{-2} = 0.1353\ \frac{V}{R}$

At $t = 4\tau$, $\quad i(t) = \frac{V}{R}e^{-4} = 0.0183\ \frac{V}{R}$

At $t = 6\tau$, $\quad i(t) = \frac{V}{R}e^{-6} = 0.0025\ \frac{V}{R}$

From above values we can observe that, the current through capacitor drops from $\frac{V}{R}$ to 0.3679 in one time constant. But after one time constant period, this rate slows down for further period of time. Ideally the current reaches zero value at infinite time.

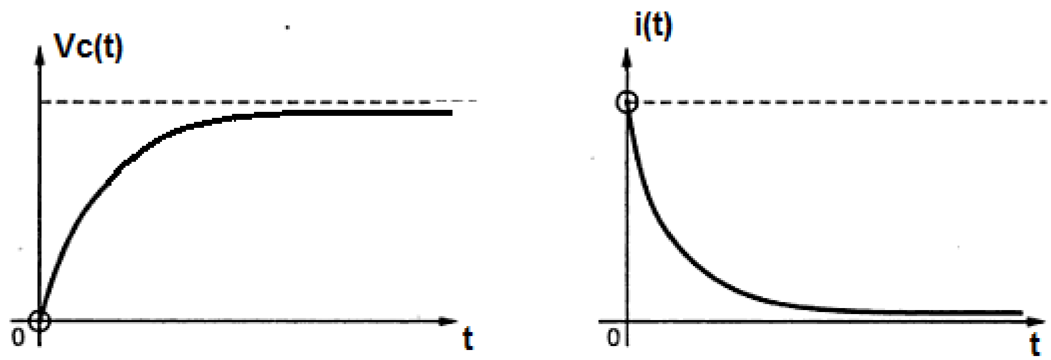


Fig. 2.5 Variation of Vc(t) and i(t) against time t

4.3.3 Transient Response of series RLC circuit for DC excitation

Consider a series RLC circuit shown in fig. 2.6, such that switch is closed at t=0 and before that switch is open for a long time.

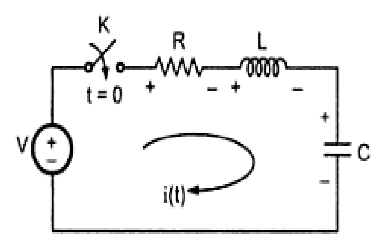


Fig. 2.6

Initial condition

At $t = 0^-$, Switch is open

$$i(0^-) = I_0 = 0$$

And also we know that inductor does not allow sudden change in current through it.

i.e $i(0^+) = i(0^-) = I_0 = 0$

Before switch is closed at $t = 0^-$, there is no current through the circuit and capacitor is also uncharged.

Current through inductor and voltage across capacitor does not change instantaneously.

$$V_C(0^+) = V_C(0^-) = V_0 = 0$$

For $t \geq 0$ the switch is closed

Apply KVL to the circuit

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)dt = V \qquad (4.7)$$

This is an integro differential equation, differentiating on both sides to get the total equation in differential form.

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = 0$$

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} = 0$$

Above equation is of quadratic expression form.

Let m_1 and m_2 be two roots for the above equation given by

$$m_1, m_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Depending on the values of R,L and C three different cases arise

Case (i)

$R^2 > \frac{4L}{C}$(Discriminant is positive) (Over damped)

The roots m_1, m_2 are real and different

The solution of this equation of the form

$$i(t) = Ae^{m_1 t} + Be^{m_2 t} (2.8)$$

Evaluate the constants A and B

To evaluate the constants A and B, we have to substitute the initial conditions

(i) At t=0, $i(0^+) = i(0^-) = I_0 = 0$

(ii) At t=0, $V_C(0^+) = V_C(0^-) = V_0 = 0$ i.e $\frac{1}{C}\int_0^t i(t)dt = 0$

At t=0, Substitute these values in the equation 4.7 and 4.8 we get

$$0 + L\frac{di(t)}{dt} + 0 = V$$

$$\frac{di(t)}{dt}\Big|_{t=0} = \frac{V}{L}$$

$$i(0) = Ae^{m_1(0)} + Be^{m_2(0)}$$

$$0 = A + B \qquad (4.9)$$

Differentiate equation 4.8

$$\frac{di(t)}{dt}\Big|_{t=0} = Am_1e^{m_1(0)} + Bm_2e^{m_2(0)}$$

$$Am_1 + Bm_2 = \frac{V}{L} \qquad (4.10)$$

From equations 4.9 and 4.10

$$A(m_1 - m_2) = \frac{V}{L}$$

$$A = \frac{V}{L(m_1 - m_2)}$$

$$B = \frac{-V}{L(m_1 - m_2)}$$

$$i(t) = \frac{V}{L(m_1 - m_2)}[e^{m_1 t} - e^{m_2 t}]$$

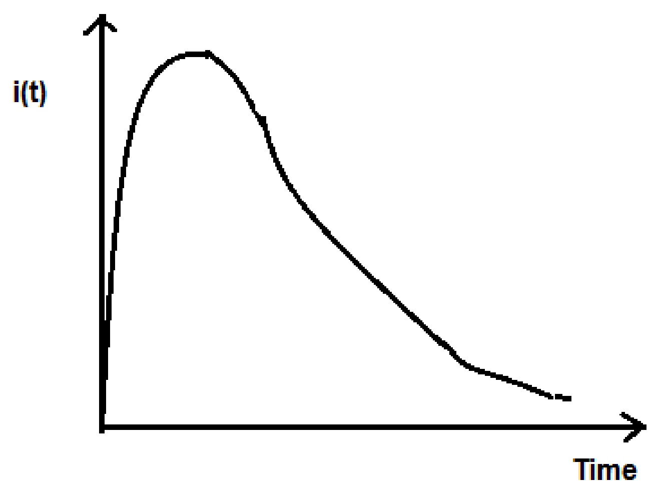


Fig 2.7 Current response in over damped case

Case (ii)

$R^2 = \frac{4L}{C}$ **(Discriminant is Zero) (Critically damped)**

The roots m_1, m_2 are real and equal.

$$m_{1,2} = -\frac{R}{2L}$$

The general solution of differential equation when roots of characteristic equation are equal is

$$i(t) = (A + Bt)e^{mt}$$

$$i(t) = (A + Bt)e^{-\frac{R}{2L}t}$$

Evaluate the constants A and B

To evaluate the constants A and B, we have to substitute the initial conditions

(i) At t=0, $i(0^+) = i(0^-) = I_0 = 0$

(ii) At t=0, $V_C(0^+) = V_C(0^-) = V_0 = 0$ i.e $\frac{1}{C}\int_0^t i(t)dt = 0$

(i) $i(0) = (A + B(0))e^{-\frac{R}{2L}(0)}$

$$i(0) = A = 0$$

(ii) $\frac{di(t)}{dt}|_{t=0} = \frac{V}{L}$

$$\frac{di(t)}{dt} = (A + Bt)\left(-\frac{R}{2L}\right)e^{-\frac{R}{2L}t} + (B)e^{-\frac{R}{2L}t}$$

$$\frac{di(t)}{dt}|_{t=0} = (A + B(0))\left(-\frac{R}{2L}\right)e^{-\frac{R}{2L}(0)} + (B)e^{-\frac{R}{2L}(0)} = \frac{V}{L}$$

$A\left(-\frac{R}{2L}\right) + (B) = \frac{V}{L}$

$B = \frac{V}{L}$ since $A = 0$

$i(t) = \frac{V}{L}t\,e^{-\frac{R}{2L}t}$

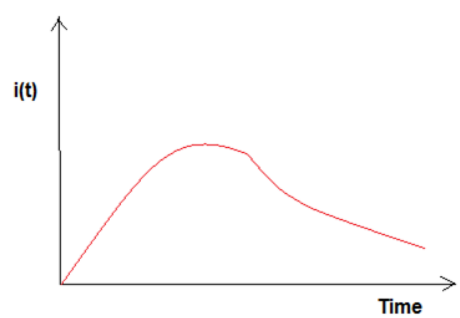


Fig 2.8 Current response in critically damped case

Case (iii)

$R^2 < \frac{4L}{C}$**(Discriminant is negative) (Under damped)**

The roots m_1, m_2 are Complex conjugates.

$$m_1 = -\alpha + j\beta$$

$$m_2 = -\alpha - j\beta$$

$$= -\frac{R}{2L} \pm \frac{1}{2L}\sqrt{R^2 - \frac{4L}{C}}$$

$$\alpha = -\frac{R}{2L}$$

$$\beta = \frac{1}{2L}\sqrt{\frac{4L}{C} - R^2}$$

The solution is

$$i(t) = Ae^{(-\alpha+j\beta)t} + Be^{(-\alpha-j\beta)t} = e^{-\alpha t}\left[Ae^{(j\beta)t} + Be^{(-j\beta)t}\right]$$

$$= e^{-\alpha t}[(A+B)\cos\beta t + j(A-B)\sin\beta t]$$

$$i(t) = e^{-\alpha t}[M\cos\beta t + N\sin\beta t] \qquad (4.11)$$

Where M=$(A+B)$ and N= $j(A-B)$

The constants can be found by using initial conditions

(i) At t=0, $i(0^+) = i(0^-) = I_0 = 0$

(ii) At t=0, $V_C(0^+) = V_C(0^-) = V_0 = 0$

i.e $\frac{1}{C}\int_0^t i(t)dt = 0$

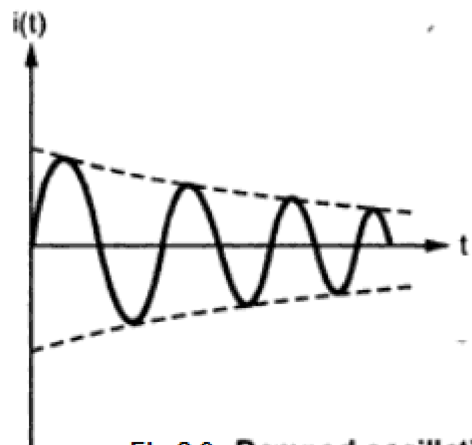


Fig.2.9 **Damped oscillations**

4.4 LAPLACE TRANSFORM

4.4.1 Introduction

Laplace transform is an alternate approach to solve transients which is easier for complex circuits.So, the transform method in general can be represented by the expression

$$f(t) \rightarrow F(s)$$

Which shows the one-to-one correspondence between the time-domain function f(t) and its frequency domain transform F(s), where $s = \sigma + j\omega$ is the complex frequency.

4.4.2 Definition of the Laplace Transform

The so called two-sided or bilateral Laplace transform of F(t) is defined as

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t) dt$$

In circuit analysis problems the forcing and response functions do not usually exist endlessly in time, but rather they are initiated at some specific instant of time selected as t=0. Thus, such functions that do not exist for t<0 can be described with the help of unit step functions as f (t)u(t). For these functions the Laplace transform defining integral is taken with the lower limit at $t = 0^{-}$.

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t) dt = \int_{0^{-}}^{\infty} e^{-st} f(t) dt$$

The latter integral defines the one-sided or unilateral Laplace transform, or simply the Laplace transform of f (t). The lower limit $t = 0^{-}$(as distinguished from t= 0 or t=0+) in a one-sided Laplace transform is taken in order to include the effect of any discontinuity at t=0, such as an impulse function and independent initial conditions such as currents in inductances $i_L(0^{-})$ and voltages across capacitances $v_C(0^{-})$.

Key Points

1. The terms "two-sided" or bilateral are used to emphasize the fact that both positive and negative times are included in the range of integration.
2. In transient analysis of electric circuits $t = 0^{-}$is denoted as the time just before the switching action, and $t = 0^{+}$as the time just after the switching action, representing radically different states of the circuit. Mathematically, $f(0^{-})$ is the limit of f (t) as t approaches zero through negative values (t<0), or the limit from the right, and $f(0^{+})$ is the limit as t approaches zero through positive values (t>0), or the limit from the left.

4.4.3 Steps in Applying the Laplace Transform:

1. Transform the circuit from the time domain to the s-domain.
2. Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any circuit analysis technique with which we are familiar.
3. Take the inverse transform of the solution and thus obtain the solution in the time domain.

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as} F(s)$
Frequency shift	$e^{-at} f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2 f}{dt^2}$	$s^2 F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3 f}{dt^3}$	$s^3 F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x)\,dx$	$\frac{1}{s} F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds} F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1 - e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.

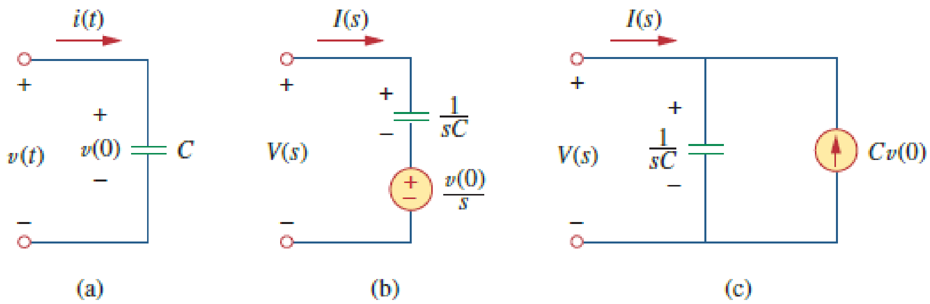


Figure
Representation of a capacitor: (a) time-domain, (b,c) *s*-domain equivalents.

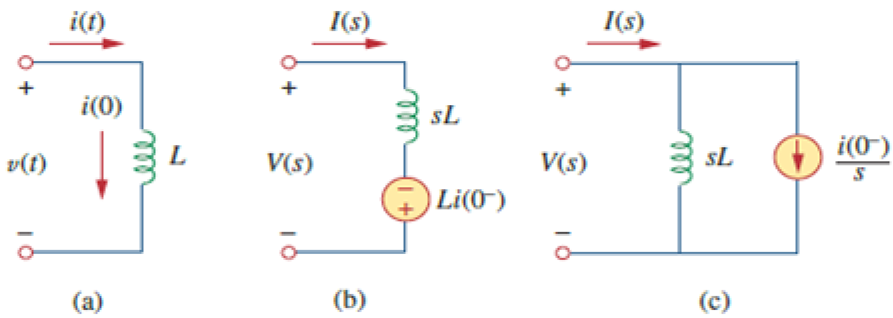


Figure
Representation of an inductor: (a) time-domain, (b,c) *s*-domain equivalents.

4.5 Application of Laplace Transforms to series RL, RC, RLC circuits with DC Excitation

4.5.1 Transient Response of series RL circuit for DC excitation

Consider series RL circuit shown in Fig.. Consider that switch closed at t=0 and before that switch is open for a long time.

Initial condition

At $t = 0^-$, Switch is open

$i(0^-) = I_0 = 0$

And also we know that inductor does not allow sudden change in current through it.

i.e $i(0^+) = i(0^-) = I_0 = 0$

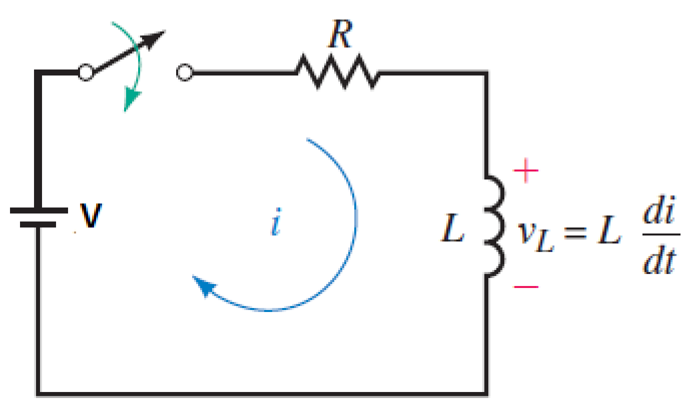


Fig.2.13

For $t \geq 0$ the switch is closed

ApplyKVL to the circuit

$Ri(t) + L\frac{di(t)}{dt} = V$

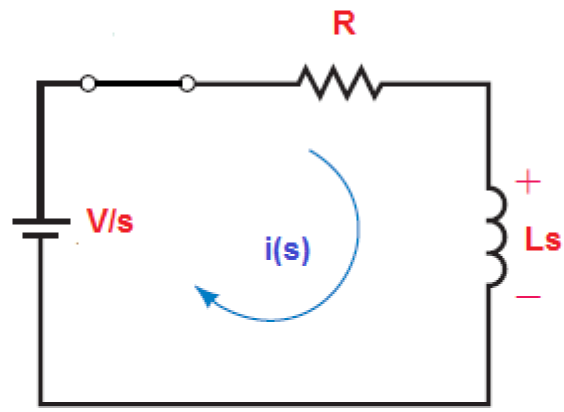


Fig.2.14

ApplyLaplace transform to the above equation

$$RI(s) + L\big(sI(s) - i(0)\big) = \frac{V}{s}$$

We know that

$$i(0^+) = i(0^-) = I_0 = 0$$

$$RI(s) + L(sI(s) - 0) = \frac{V}{s}$$

$$RI(s) + LsI(s) = \frac{V}{s}$$

$$I(s)(R + Ls) = \frac{V}{s}$$

$$I(s) = \frac{V}{s(R+Ls)}$$

Applying partial fractions

$$I(s) = \frac{A}{s} + \frac{B}{(R+Ls)} \qquad (4.18)$$

$$\frac{V}{s(R + Ls)} = \frac{A}{s} + \frac{B}{(R + Ls)}$$

$$\frac{V}{s(R + Ls)} = \frac{A(R + Ls) + Bs}{s(R + Ls)}$$

$$V = A(R + Ls) + Bs \qquad (4.19)$$

Substitute s=0 in equation 4.19

$$V = A(R + 0) + B(0)$$

$$A = \frac{V}{R}$$

Substitute $s = -\frac{R}{L}$ in equation 4.19

$$V = A\left(R + L\left(\frac{-R}{L}\right)\right) + B\left(\frac{-R}{L}\right)$$

$$B = \frac{V}{\left(\frac{-R}{L}\right)}$$

Substitute A and B values in the equation 4.18 we get

$$I(s) = \frac{\frac{V}{R}}{s} + \frac{\frac{V}{\left(\frac{-R}{L}\right)}}{(R + Ls)} = \frac{(\frac{V}{R})}{s} - \frac{VL}{RL\left(s + \frac{R}{L}\right)}$$

$$I(s) = \frac{(\frac{V}{R})}{s} - \frac{(\frac{V}{R})}{\left(s + \frac{R}{L}\right)}$$

Applying inverse Laplace transform to the above equation we get

$i(t) = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t} = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$

The voltage across the Resistance

$V_R = Ri(t)$

$$\boxed{V_R(t) = V(1 - e^{-\frac{R}{L}t})}$$

$$V_L(t) = V - V_R(t)$$

$$\boxed{V_L(t) = Ve^{-\frac{R}{L}t}}$$

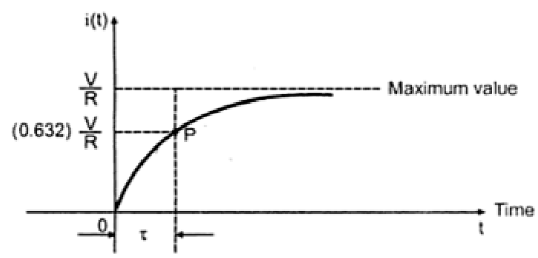


Fig.2.15

We can see that the current starts at zero and increases up to V/R with a time constant ($\tau = \frac{L}{R}$) and with the behaviour of an inverse exponential decay function. It is common to consider the current in steady-state after approximately 5τ.

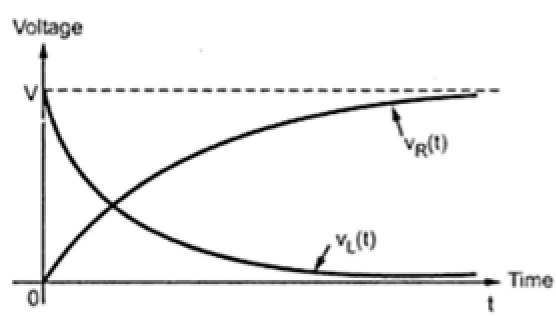


Fig.2.16 **Variation of $v_R(t)$ and $v_L(t)$ with time**

It is important to keep in mind that the current cannot change instantaneously due to the conservation of the movement in the magnetic flux associated to the inductor. In other words, the magnetic flux has to be continuous, i.e., an instant change in the current would require an infinite voltage which is obviously impossible in a real system.

4.5.2 Transient Response of series RC circuit for DC excitation

Consider series RC circuit shown in Fig2.17. Consider that switch closed at t=0 and before that switch is open for a long time.

To find transient response of driven series RC circuit means to find expression for voltage across capacitor $V_C(t)$

Initial condition

At $t = 0^-$, Switch is open

Before closing the switch active source is not presented in the circuit, so the initial voltage across capacitor is zero.

$$V_C(0^-) = V_0 = 0$$

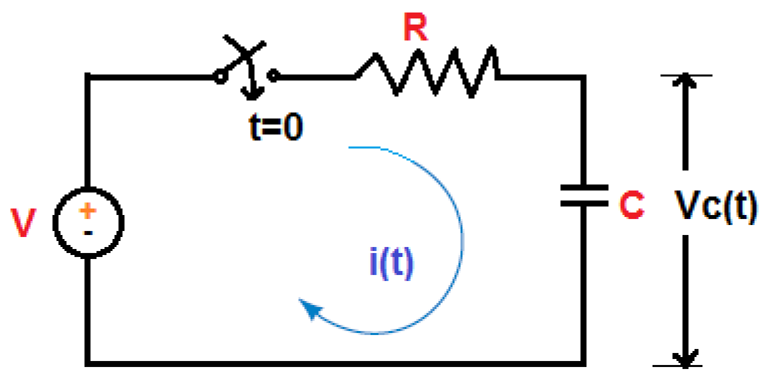


Fig.2.17

And also we know that Capacitor does not allow sudden change in voltage across it.

i.e $V_C(0^+) = V_C(0^-) = V_0 = 0$

For $t \geq 0$ the switch is closed

Now the voltage source is introduced in the circuit.

Apply KVL to the circuit

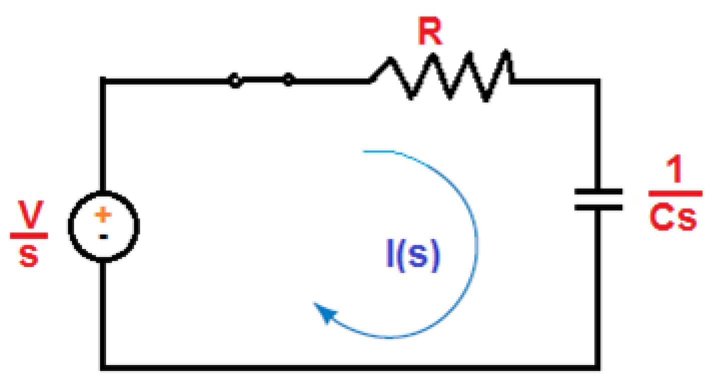


Fig.2.18

$$\frac{V}{s} = RI(s) + \frac{1}{Cs}I(s)$$

$$V = I(s)\left(Rs + \frac{1}{C}\right)$$

$$I(s) = \frac{V}{Rs + \frac{1}{C}}$$

$$I(s) = \frac{V}{R(s + \frac{1}{RC})}$$

Applying Inverse Laplace transform for the above equation we get

$$\mathrm{i(t)} = \frac{\mathrm{V}}{\mathrm{R}} \mathrm{e}^{-\frac{1}{\mathrm{RC}}\mathrm{t}}$$

$$\boxed{i(t) = \frac{V}{R} \mathrm{e}^{-\frac{1}{RC}t}}$$

$$\mathrm{V_R(t) = i(t) * R}$$

$$\mathrm{V_R(t)} = \frac{\mathrm{V}}{\mathrm{R}} \mathrm{e}^{-\frac{1}{\mathrm{RC}}\mathrm{t}} * \mathrm{R}$$

$$\boxed{V_R(t) = V \mathrm{e}^{-\frac{1}{RC}t}}$$

$$\mathrm{V_C(t) = V - V_R(t)}$$

$$\mathrm{V_C(t)} = \mathrm{V} - \mathrm{V}\mathrm{e}^{-\frac{1}{\mathrm{RC}}\mathrm{t}}$$

$$\boxed{V_C(t) = V - V e^{-\frac{\mathrm{t}}{\mathrm{RC}}}}$$

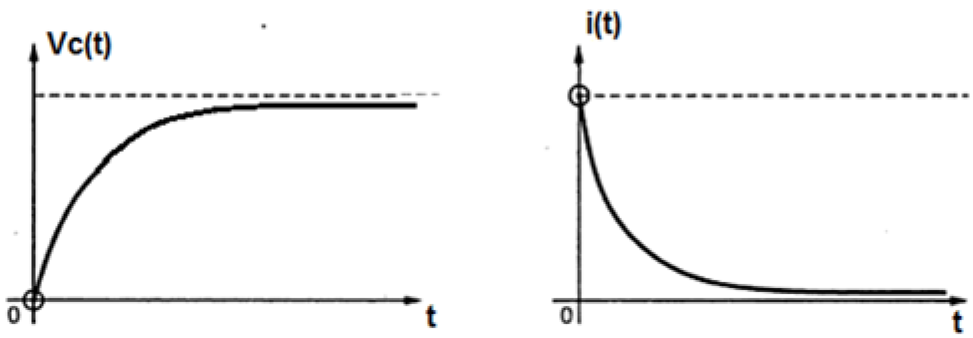


Fig.2.19 Variation of Vc(t) and i(t) against time t

2.6.3 Transient Response of series RLC circuit for DC excitation

Consider series RLC circuit shown in Fig.. Consider that switch closed at t=0 and before that switch is open for a long time.

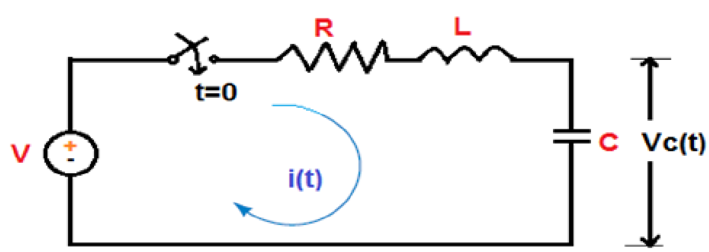


Fig.2.20

Initial condition

At $t = 0^-$, Switch is open!

$$i(0^-) = I_0 = 0$$

And also we know that inductor does not allow sudden change in current through it.

i.e $i(0^+) = i(0^-) = I_0 = 0$

Before switch is closed at $t = 0^-$, there is no current through the circuit and capacitor is also uncharged.

Current through inductor and voltage across capacitor does not change instantaneously.

$$V_C(0^+) = V_C(0^-) = V_0 = 0$$

For $t \geq 0$ the switch is closed

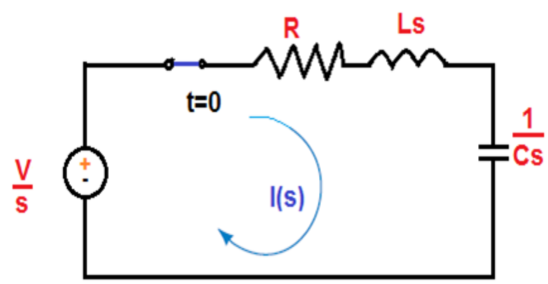


Fig.2.21

Apply KVL to the circuit

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)dt = V$$

Taking Laplace transform to the above equation

$$RI(s) + LsI(s) + \frac{1}{Cs}I(s) = \frac{V}{s}$$

$$I(s)\left(R + Ls + \frac{1}{Cs}\right) = \frac{V}{s}$$

$$I(s) = \frac{V}{s\left(R + Ls + \frac{1}{Cs}\right)}$$

$$I(s) = \frac{VC}{(RCs + LCs^2 + 1)}$$

$$I(s) = \frac{VC}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$I(s) = \frac{V}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} \qquad (4.20)$$

$$\boxed{I(s) = \frac{V}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}}$$

By taking partial fractions we get equation in the form of

$$I(s) = \left[\frac{A_1}{(s+m_1)} + \frac{A_2}{(s+m_2)}\right]$$

Then applying inverse Laplace transform to above equation we get

$$i(t) = A_1 e^{-m_1 t} + A_2 e^{-m_2 t}$$

$$\boxed{i(t) = A_1 e^{-m_1 t} + A_2 e^{-m_2 t}}$$

Here m_1 and m_2 are Roots of equation $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$$m_1 = \frac{-\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$m_2 = \frac{-\frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$m_{1,2} = -\frac{R}{2L} \pm \frac{1}{2L}\sqrt{R^2 - \frac{4L}{C}}$$

Case (i)

$R^2 > \frac{4L}{C}$(Discriminant is positive) (Over damped)

The roots m_1, m_2 are real and different

Case (ii)

$R^2 = \frac{4L}{C}$(Discriminant is Zero) (Critically damped)

The roots m_1, m_2 are real and equal.

Case (iii)

$R^2 < \frac{4L}{C}$(Discriminant is negative) (Under damped)

The roots m_1, m_2 are Complex conjugates.

UNIT - V

Transient Analysis for A.C Excitation

Objectives:

- To introduce the concept of Transients in electrical circuits.
- To study the transient behavior of RL, RC & RLC circuits for AC excitation.

Syllabus:

Transient response of series R-L, R-C and R-L-C circuits for sinusoidal excitation-Initial Conditions-Solution method using differential equation approach and Laplace transform method

Outcomes:

On completion the student should be able to:

- Analyze the transient behavior of first order and second order circuits for AC excitations using differential equation approach & Laplace transform approach.
- Obtain the transformed networks and find the response using inverse Laplace transforms.

5.1 Introduction to Transients

Transient analysis (or just transients) of electrical circuits is as important as steady-state analysis. When transients occur, the currents and voltages in some parts of the circuit may many times exceed those that exist in normal behavior and may destroy the circuit equipment in its proper operation. We may distinguish the transient behavior of an electrical circuit from its steady-state, in that during the transients all the quantities, such as currents, voltages, power and energy, are changed in time, while in steady-state they remain invariant, i.e. constant (in d.c operation) or periodical (in a.c operation) having constant amplitudes and phase angles.

The cause of transients

- change in circuit parameters and/or in circuit configurations, which usually occurs as a result of switching (commutation),
- short, and/or open circuiting,

- Change in the operation of sources etc.

The transient processes are attained by the interchange of energy, which is usually stored in the magnetic field of inductances or/and the electrical field of capacitances. Any change in energy cannot be abrupt otherwise it will result in infinite power (as the power is a derivative of energy, p=dw/dt), which is in contrast to physical reality. All transient changes, which are also called transient responses (or just responses), vanish and, after their disappearance, a new steady-state operation is established. In this respect, we may say that the transient describes the circuit behavior between two steady states: an old one, which was prior to changes, and a new one, which arises after the changes.

A few methods of transient analysis are known: the classical method, The Cauchy-Heaviside (C-H) operational method, the Fourier transformation method and the Laplace transformation method.

Comparing the classical method and the laplace transformation method it should be noted that the latter requires more knowledge of mathematics and is less related to the physical matter of transient behavior of electric circuits than the former.

Classical method of transient analysis is based on the determination of differential equations and splitting the solution into two components: natural and forced responses. The classical method is fairly complicated mathematically, but is simple in engineering practice.

5.2 Natural and Forced Responses:

Solving differential equations by the classical method, complete solution of any linear differential equation as composed of two parts: the complementary solution (or natural response) and the particular solution (or forced response). To understand these principles, let us consider a first order differential equation,

$$\frac{dv}{dt} + P(t)v = Q(t) \qquad (4.1)$$

Here Q(t) is identified as a forcing function, which is generally a function of time (or constant, if a d.c. source is applied) and P(t), is also generally a function of time, represents the circuit parameters. In our study, however, it will be a constant quantity, since the value of circuit elements does not change during the transients (indeed, the circuit parameters do change during the transients, but we may neglect this change as in many cases it is not significant).

$$v = e^{-Pt} \int Q e^{Pt} dt + A e^{-Pt} \qquad (4.2)$$

General solution can be written as

$$v = v_{PI} + v_{CF}$$

$$v_{PI} = \text{Particular integral} = e^{-Pt} \int Q e^{Pt} dt$$

$$v_{CF} = \text{Complementary Function} = A e^{-Pt}$$

In general v_{PI} may be written as a Steady state value, designated asV_{ss}. Remaining part V is called Transient portion of solution V_t.

$$V = V_{ss} + V_t$$

$$V_{PI} = V_{ss} = \text{Source response} = \text{ Steady state response} = \text{Forced response}$$

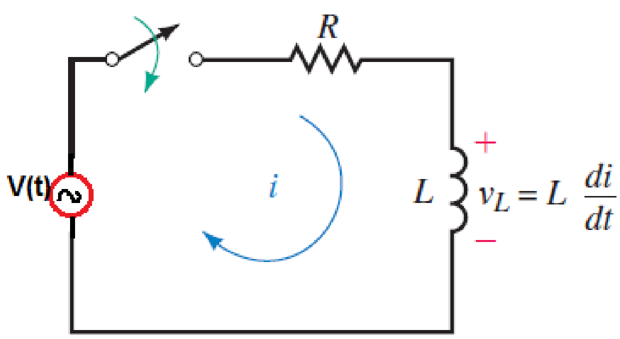
$$V_{CF} = V_t = \text{Source free response} = \text{ Transient response} = \text{Natural response}$$

Complete solution is composed of two parts. The first one, which is dependent on the forcing function Q, is the forced response (it is also called the steady state response or the particular solution or the particular integral). The second one, which does not depend on the forcing function, but only on the circuit parameters P (the types of elements, their values, interconnections, etc) and on the initial conditions A, i.e., on the “nature” of the circuit, is the natural response. It is also called the solution of the homogeneous equation, which does not include the source function and has anything but zero on its right side. Following this rule, we will solve differential equations by finding natural and forced responses separately and combining them for a complete solution.

Element	Behaviour immediately after excitiation is given $t=0^+$ instant	Behaviour as $t \to \infty$ i.e. steady state
R	R	R
L	O.C.	S.C.
I_0	I_0	S.C.
C	S.C.	O.C.
V_0 + − C	V_0 + −	V_0 + − O.C.

5.3 Transient response of series RL, RC, RLC circuit for AC excitation

5.3.1 Transient Response of series RL circuit for AC excitation



$$V(t) = V_m \sin(\omega t + \theta)$$

Consider series RL circuit shown in figure.

Consider that switch closed at t=0 and before that switch is open for a long time.

Initial condition

At $t = 0^-$, Switch is open

$i(0^-) = I_0 = 0$

And also we know that inductor does not allow sudden change in current through it.

i.e $i(0^+) = i(0^-) = I_0 = 0$

For t $\geq$ 0 the switch is closed

Apply KVL to the circuit

$$Ri(t) + L\frac{di(t)}{dt} = V_m \sin(\omega t + \theta) \qquad (5.1)$$

Rearranging

$$\left[D + \frac{R}{L}\right] i = \frac{V_m}{L}\sin(\omega t + \theta)$$

The complementary function of the above D.E is

$$i_c = Ke^{-\frac{Rt}{L}}$$

This is the transient part of the solution. The steady state solution of equation 5.1 can be obtained by assuming particular integral as

$$i_p(t) = A\ cos\omega t + Bsin\omega t$$

Substituting this solution in eq 5.1 at $\theta = 0$

$$RA\ cos\omega t + RBsin\omega t + L(-\omega Asin\omega t + \omega Bcos\omega t) = V_m \sin \omega t$$

Comparing similar terms

$$RA + \omega LB = 0$$

$$RB - \omega LA = V_m$$

Solving for A and B

$A = -\frac{\omega L V_m}{R^2+\omega^2 L^2}$ and $B = \frac{V_m R}{R^2+\omega^2 L^2}$

$$i_p(t) = -\frac{\omega L V_m}{R^2 + \omega^2 L^2}\ cos\omega t + \frac{V_m R}{R^2 + \omega^2 L^2} sin\omega t$$

$$i_p(t) = \frac{V_m}{R^2 + \omega^2 L^2}[Rsin\omega t - \omega Lcos\omega t] = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin(\omega t - \phi)$$

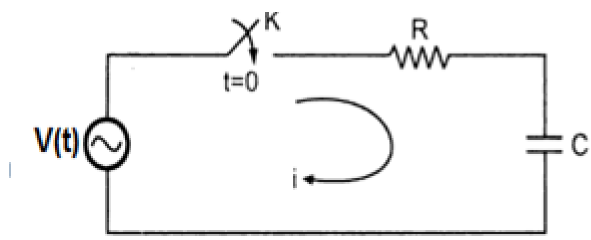
Where $tan\phi = \frac{\omega L}{R}$

The complete solution is given by

$$i(t) = Ke^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \phi)$$

K can be found from initial condition i(0)=0 and finally the complete solution.

5.3.2 Transient Response of series RC circuit for AC excitation



$$V(t) = V_m \sin(\omega t + \theta)$$

Consider series RC circuit shown in Fig..

Consider that switch closed at t=0 and before that switch is open for a long time.

To find transient response of driven series RC circuit means to find expression for voltage across capacitor $V_C(t)$.

Initial condition

At $t = 0^-$, Switch is open

Before closing the switch active source is not presented in the circuit, so the initial voltage across capacitor is zero.

$$V_C(0^-) = V_0 = 0$$

And also we know that Capacitor does not allow sudden change in voltage across it.

i.e $V_C(0^+) = V_C(0^-) = V_0 = 0$

For t $\geq$ 0 the switch is closed

Now the voltage source is introduced in the circuit.

Apply KVL to the circuit

$$\mathrm{Ri} + \frac{1}{\mathrm{C}}\int \mathrm{i}\,\mathrm{dt} = \mathrm{V_m}\sin(\omega \mathrm{t} + \theta)$$

Differentiating w.r.t time

$$R\frac{di}{dt} + \frac{i}{C} = V_m \sin(\omega t + \theta)$$

$$\frac{di}{dt} + \frac{i}{RC} = \frac{V_m \omega}{R}\cos(\omega t + \theta) \qquad (5.2)$$

The transient part of the solution is

$$\mathrm{i_t} = \mathrm{Ke}^{-\frac{\mathrm{t}}{\mathrm{RC}}}$$

This is the transient part of the solution. The steady state solution of equation 5.2 can be obtained by assuming particular integral as

$$i_p(t) = A\,cos\omega t + Bsin\omega t$$

Substituting this solution in eq 5.2 at $\theta = 0$

$$R(-\omega A sin\omega t + \omega B cos\omega t) + \frac{1}{C}(A\,cos\omega t + Bsin\omega t) = \frac{\omega \mathrm{V_m}}{R}\cos \omega \mathrm{t}$$

Comparing similar terms

$$-\omega RA + \frac{B}{C} = 0$$

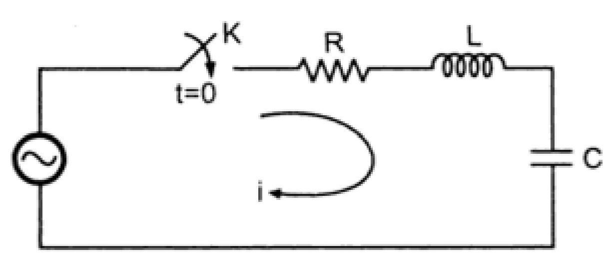
$$R\omega B + \frac{A}{C} = \frac{\omega \mathrm{V_m}}{R}$$

Solving for A and B and finally the complete solution is given by

$$i(t) = Ke^{-\frac{t}{RC}} + \frac{V_m}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}\sin(\omega t + \phi) \qquad (5.3)$$

K can be found from initial condition.

5.3.3 Transient Response of series RLC circuit for AC excitation



$$V(t) = V_m \sin(\omega t + \theta)$$

Consider series RLC circuit shown in Fig.. Consider that switch closed at t=0 and before that switch is open for a long time.

Initial condition

At $t = 0^-$, Switch is open

$$i(0^-) = I_0 = 0$$

And also we know that inductor does not allow sudden change in current through it.

Current through inductor and voltage across capacitor does not change instantaneously.

Before switch is closed at $t = 0^-$, there is no current through the circuit and capacitor is also uncharged.

$$i(0^+) = i(0^-) = I_0 = 0$$

$$V_C(0^+) = V_C(0^-) = V_0 = 0$$

For $t \geq 0$ the switch is closed

Apply KVL to the circuit

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt = V_m \sin(\omega t + \theta) \qquad (5.4)$$

This is an integro differential equation, differentiating on both sides to get total equation in differential equation.

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \omega V_m \cos(\omega t + \theta)$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{\omega V_m}{L}\cos(\omega t + \theta)$$

Replace $\frac{d^2}{dt^2}$ with D^2 and $\frac{d}{dt}$ with D we get characteristic equation or auxiliary equation

$$\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right] i = 0$$

The Transient solution

The Transient solution of this characteristic equation of the form

$$i(t) = Ae^{m_1 t} + Be^{m_2 t} \qquad (5.5)$$

Where, A and B are constants and m_1, m_2 are roots of the characteristic equation. The response of the circuit depends upon the nature of the roots of the characteristic equation.

The roots are,

$$m_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)^2}$$

S.No	**Condition**	**Nature of the roots**
1	$R^2 - \frac{4L}{C} > 0$ or $R^2 > \frac{4L}{C}$	Real and different roots
2	$R^2 - \frac{4L}{C} = 0$ or $R^2 = \frac{4L}{C}$	Real and equal roots
3	$R^2 - \frac{4L}{C} < 0$ or $R^2 < \frac{4L}{C}$	Complex conjugates roots

$$m_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)^2}$$

$$m_{1,2} = -\frac{R}{2L} \pm \frac{1}{2L}\sqrt{R^2 - \frac{4L}{C}}$$

Let

$$\alpha = -\frac{R}{2L}\,,\ \beta = \frac{1}{2L}\sqrt{\frac{4L}{C} - R^2}$$

S.No	Nature of the roots	Transient part of solution
1	Real and different roots	$i_t = Ae^{-m_1 t} + Be^{-m_2 t}$
2	Real and equal roots	$i_t = (A + Bt)e^{-mt}$
3	Complex conjugates roots	$i_t = e^{-\alpha t}[M\cos\beta t + N\sin\beta t]$ Or $i_t = e^{-\alpha t}[A\cos\beta t + B\sin\beta t]$

The Steady state solution

The steady state part of solution is given by

$$i_s = \frac{V_m}{Z}[\sin(\omega t \pm \phi)]$$

Where $tan\,\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$

Note: The steady state current in the network lags by an voltage by an angle if $X_L > X_C$ and leads the voltage $X_C > X_L$.

The total solution

$$i(t) = i_s + i_t$$

$$i(t) = \frac{V_m}{Z}[\sin(\omega t \pm \phi)] + Ae^{-m_1 t} + Be^{-m_2 t} \qquad (5.6)$$

Calculation of constants A and B

Constants A and B are calculated using initial conditions

(i) At t=0, $i(0^+) = i(0^-) = I_0 = 0$

(ii) At t=0, $V_C(0^+) = V_C(0^-) = V_0 = 0$ i.e $\frac{1}{C}\int_0^t i(t)dt = 0$

$$\frac{di}{dt} = \frac{V_m}{L}\sin(\theta)$$

Assignment-Cum-Tutorial Questions
SECTION-A

1. If an RL circuit having angle ψ is switched in when the applied sinusoidal voltage wave is passing through an angle θ, there will be no switching transient if

a) $\psi - \theta = 0$ b) $\psi - \theta = 90$ c) $\psi + \theta = 0$ d) $\psi + \theta = 90$

2. A two terminal black box contains one of the RLC elements. The black box is connected to a 220 volts ac supply. The current through the source is I. When a capacitance of 0.1 F is inserted in series between the source and the box, the current through the source is 2I. The element is

a) a resistance b) an inductance
c) a capacitance of 0.5 F d) not identifiable on the basis of the given data

3. A 10 Ω resistor, a 1 H inductor and 1 μF capacitor are connected in parallel. The combination is driven by a unit step current. Under the steady state condition, the source current flows through.

a) the Resistor b) the Capacitor c) the inductor d) All the three elements

4. If the Laplace transform of the voltage across a capacitor of value of ½ F is

The value of the current through the capacitor at $t = 0^+$ is

$$V(s) = \frac{S+1}{S^3+S^2+S+1}$$

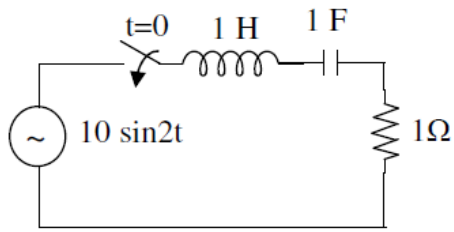
a) 0 A b) 2 A (c) (1/2) A (d) 1 A

5. A ramp voltage, v(t) = 100 t volts, is applied to an RC differentiating circuit with R = 5kΩ and C = 4 μF. The maximum output voltage is……………………

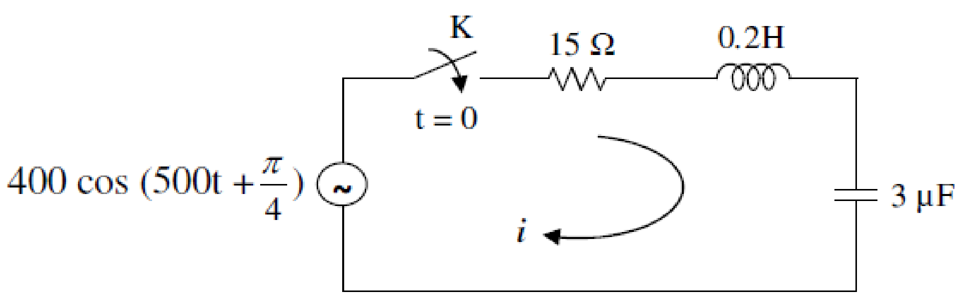
SECTION-B

1. A series R-L circuit has R=20 ohms and L=8 H. The circuit is connected across a AC voltage source of 120 V at t=0. Calculate the time at which the voltage drops across R and L are the same.

2. Find the current in the circuit shown in fig. for t>0. At t=0- the network was unenergized.



3. In the Figure , determine complete solution for current, when switch K is closed at t = 0 for applied voltage $v(t) = 400\,cos\,(500t + \pi/4)$. Derive the expression for the current



4. Derive mathematical expression for the transient response of series R-L circuit for an excitation of V Cos(ωt +φ).

5. Derive mathematical expression for the transient response of series R-C circuit for an excitation of V Cos(ωt +φ).

6. Derive mathematical expression for the transient response of series R-L-C circuit for an excitation of V Cos(ωt +φ).

SECTION-C

1. A unit step voltage is applied at t = 0 to a series RL circuit with zero initial conditions.

   a) It is possible for the current to be oscillatory.

   b) The voltage across the resistor at t = 0+ is zero.

   c) The energy stored in the inductor in the steady state is zero.

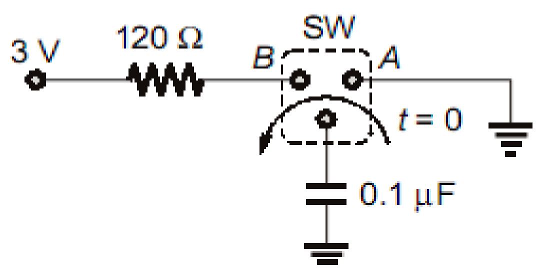
   d) The resistor current eventually falls to zero

2. The time-domain behavior of an RL circuit is represented by

$$L\frac{di}{dt} + Ri = V_o(1 - Be^{-\left(\frac{R}{L}t\right)}\sin(t))u(t) .$$

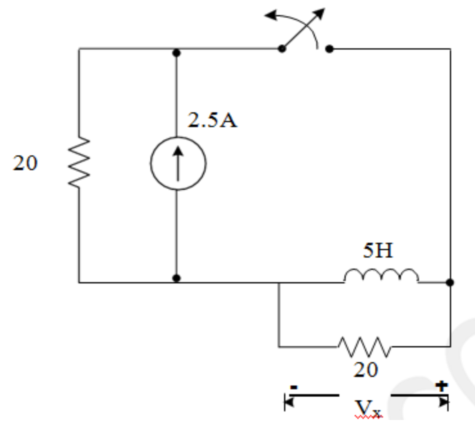
For an initial current of i(0)= Vo/R What is the steady state value of the current.

3. In the circuit shown, the switch SW is thrown from position A to position B at time t = 0. What is the energy (in μJ) taken from the 3 volts source to charge the 0.1μF capacitor from 0 to 3 volts.

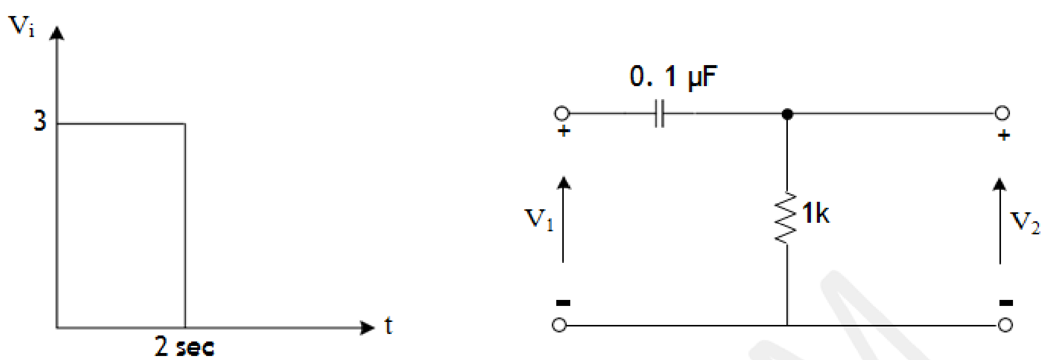


4. In the figure, the switch was closed for a long time before opening at t = 0.

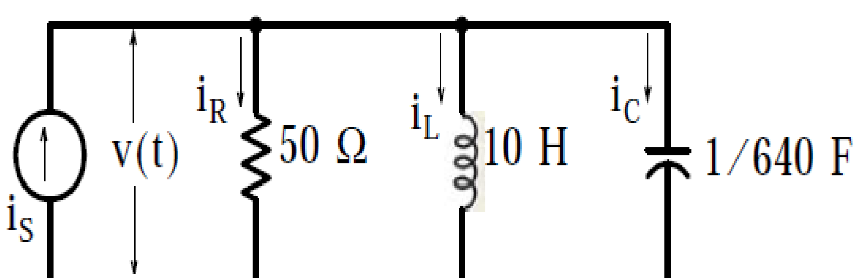
   The voltage Vx at t = 0^{+} is



5. A square pulse of 3 volts amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage V2 at time t = 2 sec is

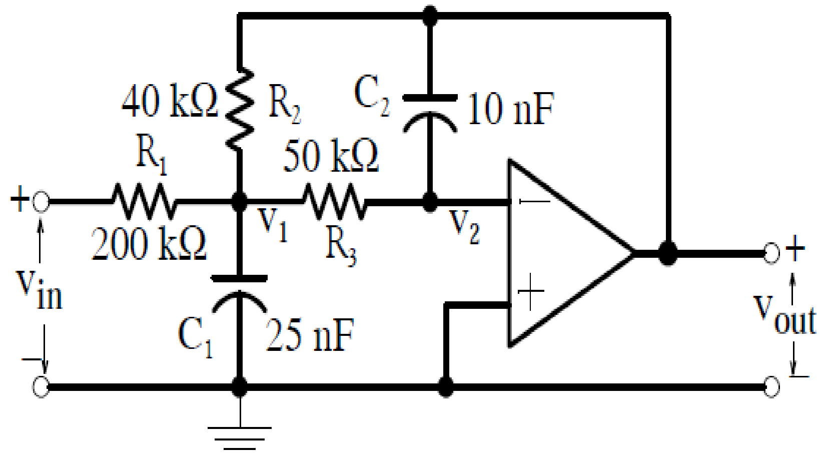


6. For the circuit of figure, $I_L(0)$=2 A and $V_C(0)$=5V. Sketch V(t) for t >0



$$i_S = 20\sin(6400t + 90^\circ)u_0(t)\ \text{A}$$

7. The circuit of Figure 1.26 a known as a Multiple Feed Back (MFB) active low-pass filter. For this circuit, the initial conditions are Vc1=Vc2=0. Compute and sketch for .



$v_{in}(t) = (6.25 \cos 6280t)u(t)$ V

UNIT - VI

Two port Networks

Objectives:

- To study the relationship between the input and output voltages and currents and define different sets of two port parameters.
- To study the relationship between different two port networks and interconnection of two port networks.

Syllabus:

Introduction to Two port networks- Z, Y, ABCD and hybrid parameters and their relations. Cascaded networks.

Outcomes:

On completion, the student should be able to:

- Determine Z, Y, ABCD & h parameters for two port networks.
- Derive the relationship between different two port parameters.
- Interconnect different two port networks and obtain the relation between the input and output quantities of the combined two-port network.

6.1 Introduction:

A two-port network has two pairs of terminals, one pair at the input known as input port and one pair at the output known as output port as shown in figure: 6.1. There are four variables V_1, V_2, I_1 and I_2 associated with a two port network. Two of these variables can be expressed in terms of the other two variables. Thus, there will be two dependent variables and two independent variables. The number of possible combinations generated by four variables taken two at a time is ${}^4C_2$, i.e., six. There are six possible sets of equations describing a two-port network.

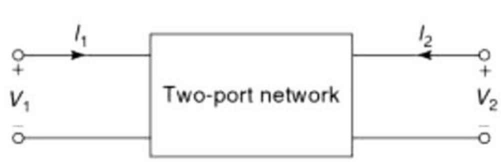


Figure: 6.1 Two-port network

6.2 Two-Port Parameters:

Parameter	Variables		Equation
	Express	In terms of	
Open-Circuit Impedance	V_1, V_2	I_1, I_2	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$
Short-Circuit Admittance	I_1, I_2	V_1, V_2	$I_1 = Y_{11}V_1 + Y_{12}V_2$ $I_2 = Y_{21}V_1 + Y_{22}V_2$
Transmission	V_1, I_1	V_2, I_2	$V_1 = A\ V_2 - B\ I_2$ $I_1 = C\ V_2 - D\ I_2$
Inverse Transmission	V_2, I_2	V_1, I_1	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$
Inverse Hybrid	I_1, V_2	V_1, I_2	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$

6.3 Open-Circuit Impedance Parameters (Z Parameters)

The Z parameters of a two-port network may be defined by expressing two-port voltages V_1 and V_2in terms of two-port currents I_1 andI_2.

$$(V_1, V_2) = f(I_1, I_2)$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

In matrix form, we can write $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z][I]$$

The individual Z parameters for a given network can be defined by setting each of the port currents equal to zero.

Case 1: When the output port is open-circuited, i.e., $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}/I_2 = 0$$

Where Z_{11} is the driving-point impedance with the output port open-circuited. It is also called open-circuit input impedance.

Similarly, $$Z_{21} = \frac{V_2}{I_1}/I_2 = 0$$

Where Z_{21} is the transfer impedance with the output port open-circuited. It is also called open-circuit forward transfer impedance.

Case 2: When the input port is open-circuited, i.e., $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2}/I_1 = 0$$

Where Z_{12} is the transfer impedance with the input port open-circuited. It is also called open-circuit reverse transfer impedance.

Similarly, $$Z_{22} = \frac{V_2}{I_2}/I_1 = 0$$

Where Z_{22} is the open-circuit driving-point impedance with the input port open-circuited. It is also called open-circuit output impedance.

As these impedance parameters are measured with either the input or output port open-circuited, these are called open-circuit impedance parameters.

The equivalent circuit of the two-port network in terms of Z parameters is shown in figure: 6.3

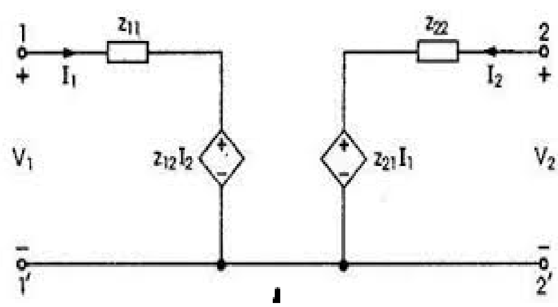


Figure: 6.3 Equivalent circuit of the two-port network in terms of Z parameter.

Condition for Reciprocity:

If Z_{12}= Z_{21} ,the network is said to be reciprocal network.

Condition for Symmetry:

If Z_{11}=Z_{22}, the network is said to be symmetrical network.

6.4 Short-Circuit Admittance Parameters (Y Parameters)

The Y parameters of a two-port network may be defined by expressing two-port currents I_1 and I_2 in terms of the two-port voltages V_1 and V_2.

$$(I_1, I_2) = f(V_1, V_2)$$
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In matrix form, we can write $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$$[I] = [Y][V]$$

The individual Y parameters for a given network can be defined by setting each of the port voltages equal to zero.

Case 1: When the output port is short-circuited, i.e., V_2 =0

$$Y_{11} = \frac{I_1}{V_1} / V_2 = 0$$

Where Y_{11} is the driving-point admittance with the output port short-circuited. It is also called short-circuit input admittance.

Similarly, $Y_{21} = \frac{I_2}{V_1} / V_2 = 0$

Where Y_{21} is the transfer admittance with the output port short-circuited. It is also called short-circuit forward transfer admittance.

Case 2: When the input port is short-circuited, i.e., V_1 =0

$$Y_{12} = \frac{I_1}{V_2} / V_1 = 0$$

Where Y_{12} is the transfer admittance with the input port short-circuited. It is also called short-circuit reverse transfer admittance.

Similarly, $Y_{22} = \frac{I_2}{V_2} / V_1 = 0$

Where Y_{22} is the short-circuit driving-point admittance with the input port short-circuited. It is also called short-circuit output admittance.

As these admittance parameters are measured with either input or output port short-circuited, these are called short-circuit admittance parameters.

The equivalent circuit of the two-port network in terms of Y parameters is shown in figure: 6.4

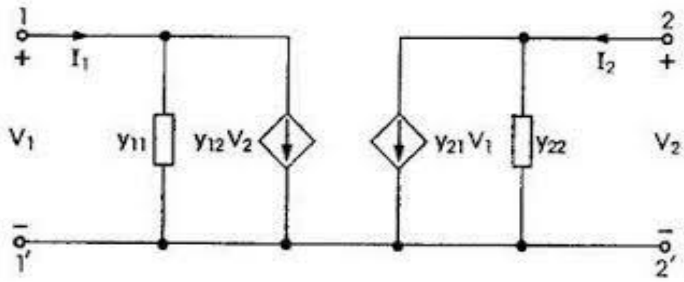


Figure: 6.4 Equivalent circuit of the two-port network in terms of Y-parameters

Condition for Reciprocity:

If $Y_{12} = Y_{21}$,the network is said to be reciprocal network.

Condition for Symmetry:

If $Y_{11} = Y_{22}$, the network is said to be symmetrical network.

6.5 Transmission Parameters (ABCD Parameters)

The transmission parameters or chain parameters or ABCD parameters serve to relate the voltage and current at the input port to voltage and current at the output port.

In equation form,

$$(V_1, I_1) = f(V_2, -I_2)$$
$$V_1 = A\,V_2 - B\,I_2$$
$$I_1 = C\,V_2 - D\,I_2$$

Here, the negative sign is used with I_2 and not for parameters B and D. The reason the current I_2 carries a negative sign is that in transmission field, the output current is assumed to be coming out of the output port instead of going into the port.

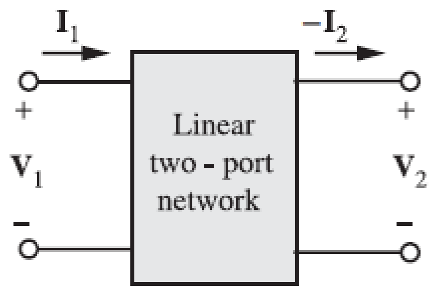


Figure: 6.5 Terminal variables used to define ABCD parameters

In matrix form, we can write $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

Where matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called transmission matrix.

For a given network, these parameters are determined as follows:

Case 1 When the output port is open-circuited, i.e., I_2=0

$$A = \frac{V_1}{V_2} / \, I_2 = 0$$

Where A is the reverse voltage gain with the output port open-circuited.

Similarly, $C = \frac{I_1}{V_2} / \, I_2 = 0$

Where C is the transfer admittance with the output port open-circuited.

Case 2 When the output port is short-circuited, i.e., V_2=0

$$B = -\frac{V_1}{I_2} / \, V_2 = 0$$

Where B is the transfer impedance with the output port short-circuited.

Similarly, $D = -\frac{I_1}{I_2} / \, V_2 = 0$

Where D is the reverse current gain with the output port short-circuited.

Condition for Reciprocity:

If AD-BC=1, the network is said to be reciprocal network.

Condition for Symmetry:

If A=D, the network is said to be symmetrical network.

6.6 Hybrid Parameters (h Parameters)

The hybrid parameters of a two-port network may be defined by expressing the voltage of input port V_1 and current of output port I_2 in terms of current of input port I_1 and voltage of output port V_2.

$$(V_1, I_2) = f(I_1, V_2)$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

In matrix form, we can write $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

These parameters are particularly important in transistor circuit analysis.

Case 1 When the output port is short-circuited, i.e., V_2=0

$$h_{11} = \left.\frac{V_1}{I_1}\right/ V_2 = 0$$

Where h_{11} is called as short-circuit input impedance.

Similarly, $h_{21} = \left.\frac{I_2}{I_1}\right/ V_2 = 0$

Where h_{21} is called as short-circuit forward current gain.

Case 2 When the input port is open-circuited, i.e.,I_1=0

$$h_{12} = \left.\frac{V_1}{V_2}\right/ I_1 = 0$$

Where h_{12} is called as open circuit reverse voltage gain.

Similarly, $h_{22} = \left.\frac{I_2}{V_2}\right/ I_1 = 0$

Where h_{22} is called as open-circuit output admittance.
Since h parameters represent dimensionally impedance, admittance, voltage gain and current gain, these are called hybrid parameters.

The equivalent circuit of the two-port network in terms of hybrid parameters is shown in figure: 6.6

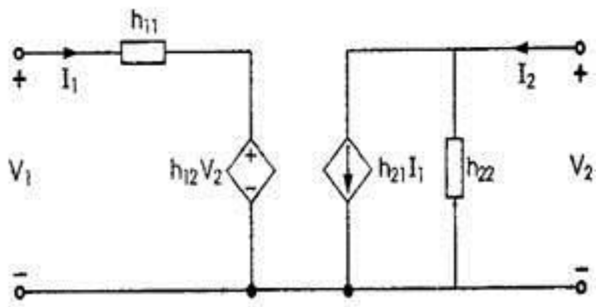


Figure: 6.6 Equivalent circuit of the two-port network in terms of h-parameters

Condition for Reciprocity:

If $h_{21} = -h_{12}$, the network is said to be reciprocal network.

Condition for Symmetry:

If $h_{11}h_{22} - h_{12}h_{21} = 1$ ($\Delta h = 1$), the network is said to be symmetrical network.

6.7 Inter-Relationships between the Parameters:

When it is required to find out two or more parameters of a particular network then finding each parameter will be tedious. But if we find a particular parameter then the other parameters can be found if the inter-relationship between them is known.

1. Z-parameters in terms of other parameters:

a) Z-parameters in terms of Y-parameters:

We known that

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

By Cramer's rule, $V_1 = \dfrac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \dfrac{Y_{22}I_1 - Y_{12}I_2}{Y_{11}Y_{22} - Y_{12}Y_{21}} = \dfrac{Y_{22}}{\Delta Y}I_1 - \dfrac{Y_{12}}{\Delta Y}I_2$

Where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

Comparing with $V_1 = Z_{11}I_1 + Z_{12}I_2$

$$Z_{11} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

Also, $V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\Delta Y}$

$$= \frac{Y_{11}}{\Delta Y} I_2 - \frac{Y_{21}}{\Delta Y} I_1$$

Comparing with $V_2 = Z_{21} I_1 + Z_{22} I_2$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y}$$

b) Z-parameters in terms of ABCD parameters:

We know that $V_1 = A\, V_2 - B\, I_2$

$I_1 = C\, V_2 - D\, I_2$

Rewriting the second equation,

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

Comparing with $V_2 = Z_{21} I_1 + Z_{22} I_2$

$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C}$$

Also, $V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - BI_2$

$$= \frac{A}{C} I_1 + \left[\frac{AD}{C} - B\right] I_2$$

$$= \frac{A}{C} I_1 + \left[\frac{AD - BC}{C}\right] I_2$$

Comparing with $V_1 = Z_{11} I_1 + Z_{12} I_2$

$$Z_{11} = \frac{A}{C}$$

$$Z_{12} = \frac{AD - BC}{C}$$

c) Z-parameters in terms of A'B'C'D' parameters:

We know that $V_2 = A'V_1 - B'I_1$
$I_2 = C'V_1 - D'I_1$

Rewriting the second equation,

$$V_1 = \frac{D'}{C'} I_1 + \frac{1}{C'} I_2$$

Comparing with $V_1 = Z_{11} I_1 + Z_{12} I_2$

$$Z_{11} = \frac{D'}{C'}$$

$$Z_{12} = \frac{1}{C'}$$

Also, $V_2 = A' \left[\frac{D'}{C'} I_1 + \frac{1}{C'} I_2\right] - B' I_1 = \left[\frac{A'D' - B'C'}{C'}\right] I_1 + \frac{A'}{C'} I_2$

Comparing with $V_2 = Z_{21} I_1 + Z_{22} I_2$

$$Z_{21} = \left[\frac{A'D' - B'C'}{C'}\right]$$

$$Z_{22} = \frac{A'}{C'}$$

d) Z-parameters in terms of Hybrid parameters:

We know that

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting second equation,

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

Comparing with $V_2 = Z_{21} I_1 + Z_{22} I_2$

$$Z_{21} = -\frac{h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

Also, $$V_1 = h_{11}I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2\right]$$

$$= h_{11}I_1 + \frac{h_{12}}{h_{22}}I_2 - \frac{h_{12}h_{21}}{h_{22}}I_1$$

$$= \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right]I_1 + \frac{h_{12}}{h_{22}}I_2$$

Comparing with $$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

2. Y-parameters in terms of other parameters:

a) Y-parameters in terms of Z-parameters:

We known that $$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

By Cramer's rule,

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}}$$

$$= \frac{Z_{22}V_1 - Z_{12}V_2}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \frac{Z_{22}}{\Delta Z}V_1 - \frac{Z_{12}}{\Delta Z}V_2$$

Where $$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

Comparing with $$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z}$$

$$Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

Also, $I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\Delta Z}$

$$= \frac{Z_{11}V_2 - Z_{12}V_1}{\Delta Z}$$

$$= -\frac{Z_{21}}{\Delta Z}V_1 + \frac{Z_{11}}{\Delta Z}V_2$$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z}$$

b) Y-parameters in terms of ABCD parameters:

We know that $V_1 = AV_2 - BI_2$

$$I_1 = CV_2 - DI_2$$

Rewriting the first equation, $I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y_{21} = -\frac{1}{B}$$

$$Y_{22} = \frac{A}{B}$$

Also, $I_1 = CV_2 - D\left[-\frac{1}{B}V_1 + \frac{A}{B}V_2\right]$

$$= \frac{D}{B}V_1 + \left[\frac{BC - AD}{B}\right]V_2$$

Comparing with $I_1 = Y_{11}V_1 + Y_{12}V_2$

$$Y_{11} = \frac{D}{B}$$

$$Y_{12} = \frac{BC - AD}{B}$$

c) Y-parameters in terms of A'B'C'D' parameters:

We know that $V_2 = A' V_1 - B' I_1$
$I_2 = C' V_1 - D' I_1$

Rewriting the first equation, $I_1 = \frac{A'}{B'}V_1 - \frac{1}{B'}V_2$

Comparing with $I_1 = Y_{11}V_1 + Y_{12}V_2$

$$Y_{11} = \frac{A'}{B'}$$
$$Y_{12} = -\frac{1}{B'}$$

Also, $I_2 = C'V_1 - D'\left[\frac{A'}{B'}V_1 - \frac{1}{B'}V_2\right]$

$$= -\left[\frac{A'D' - B'C'}{B'}\right]V_1 + \frac{D'}{B'}V_2$$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y_{21} = -\frac{A'D' - B'C'}{B'}$$
$$Y_{22} = \frac{D'}{B'}$$

d) Y-parameters in terms of Hybrid parameters:

We know that,

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Rewriting the first equation,

$$I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2$$

Comparing with $I_1 = Y_{11}V_1 + Y_{12}V_2$

$$Y_{11} = \frac{1}{h_{11}}$$
$$Y_{12} = -\frac{h_{12}}{h_{11}}$$

Also, $\quad I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2$

$$= \frac{h_{21}}{h_{11}}V_1 + \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right]V_2$$

Comparing with $\quad I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y_{21} = \frac{h_{21}}{h_{11}}$$

$$Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}$$

3. ABCD parameters in terms of other parameters:

a) ABCD parameters in terms of *Z*-parameters:

We know that

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Rewriting the second equation,

$$I_1 = \frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2$$

Comparing with, $\quad I_1 = CV_2 - DI_2$

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Also, $\quad V_1 = Z_{11}\left[\frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2\right] + Z_{12}I_2$

$$= \frac{Z_{11}}{Z_{21}}V_2 - \frac{Z_{22}Z_{11}}{Z_{21}}I_2 + Z_{12}I_2$$

$$= \frac{Z_{11}}{Z_{21}}V_2 - \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right]I_2$$

Comparing with, $\quad V_1 = AV_2 - BI_2$

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

b) ABCD parameters in terms of Y-parameters:

We know that

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Rewriting the second equation,

$$V_1 = -\frac{Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2$$

Comparing with $V_1 = AV_2 - BI_2$

$$A = -\frac{Y_{22}}{Y_{21}}$$
$$B = -\frac{1}{Y_{21}}$$

Also, $I_1 = Y_{11}\left[-\frac{Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2\right] + Y_{12}V_2$

$$= \left[\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right]V_2 + \frac{Y_{11}}{Y_{21}}I_2$$

Comparing with, $I_1 = CV_2 - DI_2$

$$C = \left[\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right] = -\frac{\Delta Y}{Y_{21}}$$
$$D = -\frac{Y_{11}}{Y_{21}}$$

c) ABCD parameters in terms of hybrid parameters:

We know that, $V_1 = h_{11}I_1 + h_{12}V_2$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Rewriting the second equation, $I_1 = -\frac{h_{22}}{h_{21}}V_2 + \frac{1}{h_{21}}I_2$

Comparing with, $I_1 = CV_2 - DI_2$

$$C = -\frac{h_{22}}{h_{21}}$$

$$D = -\frac{1}{h_{21}}$$

Also, $$V_1 = h_{11}\left[\frac{1}{h_{21}}I_2 - \frac{h_{22}}{h_{21}}V_2\right] + h_{12}V_2$$

$$= \left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right]V_2 + \frac{h_{11}}{h_{21}}I_2$$

Comparing with $$V_1 = AV_2 - BI_2$$

$$A = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}$$

$$B = -\frac{h_{11}}{h_{21}}$$

4. Hybrid parameters in terms of other parameters:

a) Hybrid parameters in terms of Z-parameters:

We know that
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Rewriting the second equation, $I_2 = -\frac{Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2$

Comparing with $$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

Also, $$V_1 = Z_{11}I_1 + Z_{12}\left[-\frac{Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2\right]$$

$$= \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}\right]I_1 + \frac{Z_{12}}{Z_{22}}V_2$$

Comparing with $$V_1 = h_{11}I_1 + h_{12}V_2$$
$$h_{11} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} = \frac{\Delta Z}{Z_{22}}$$
$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

b) Hybrid parameters in terms of Y-parameters:

We know that
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Rewriting the first equation, $V_1 = \frac{1}{Y_{11}}I_1 - \frac{Y_{12}}{Y_{11}}V_2$

Comparing with $$V_1 = h_{11}I_1 + h_{12}V_2$$
$$h_{11} = \frac{1}{Y_{11}}$$
$$h_{12} = -\frac{Y_{12}}{Y_{11}}$$

Also, $I_2 = Y_{21}\left[\frac{1}{Y_{11}}I_1 - \frac{Y_{12}}{Y_{11}}V_2\right] + Y_{22}V_2$

$$= \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}\right]V_2 + \frac{Y_{21}}{Y_{11}}I_1$$

Comparing with $$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{22} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} = \frac{\Delta Y}{Y_{11}}$$
$$h_{21} = \frac{Y_{21}}{Y_{11}}$$

c) Hybrid parameters in terms of ABCD parameters:

We know that
$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Rewriting the second equation, $I_2 = -\frac{1}{D}I_1 + \frac{C}{D}V_2$

Comparing with $$I_2 = h_{21}I_1 + h_{22}V_2$$
$$h_{21} = -\frac{1}{D}$$

$$h_{22} = \frac{C}{D}$$

Also, $\quad V_1 = AV_2 - B\left[-\frac{1}{D}I_1 + \frac{C}{D}V_2\right]$

$$= \frac{B}{D}I_1 + \left[\frac{AD - BC}{D}\right]V_2$$

Comparing with $\quad V_1 = h_{11}I_1 + h_{12}V_2$

$$h_{11} = \frac{B}{D}$$

$$h_{12} = \frac{AD-BC}{D} = \frac{\Delta T}{D}$$

Inter-relationship between parameters:

INTER-RELATIONSHIP BETWEEN PARAMETERS

	z	y	T	h
z	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
y	$\begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{22}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$
T	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
h	$\begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

$\Delta z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta T = AD - BC$

6.8 Interconnection of two-port networks:

Interconnection of two-port networks, namely, cascade, parallel, series-parallel and parallel-series are discussed below and the relation between the input and output quantities of the combined two-port networks is derived.

6.8.1 Cascade Connection:

Transmission Parameter Representation:

Figure: 6.8.1 shows two-port networks connected in cascade. In the cascade connection, the output port of the first network becomes the input port of the second network. Since it is assumed that input and output currents are positive when they enter the network, we have

$$I_1^{'} = -I_2$$

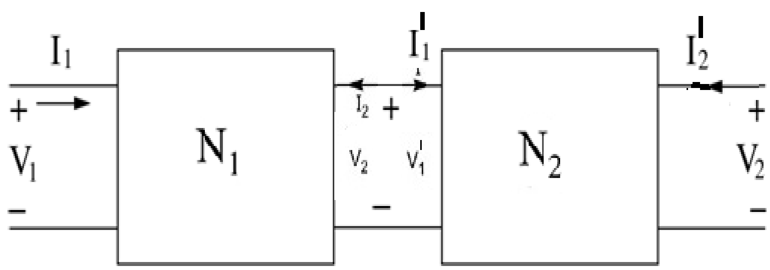


Figure: 6.8.1 Cascade Connection

Let A_1, B_1, C_1, D_1 be the transmission parameters of the network N_1 and A_2, B_2, C_2, D_2 be the transmission parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 B_1 \\ C_1 D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{———— (i)}$$

For the network N_2,

$$\begin{bmatrix} V_1^{'} \\ I_1^{'} \end{bmatrix} = \begin{bmatrix} A_2 B_2 \\ C_2 D_2 \end{bmatrix} \begin{bmatrix} V_2^{'} \\ -I_2^{'} \end{bmatrix}$$

Since $V_1^{'} = V_2$ and $I_2^{'} = -I_2$, we can write

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 B_2 \\ C_2 D_2 \end{bmatrix} \begin{bmatrix} V_2^{'} \\ -I_2^{'} \end{bmatrix} \text{———— (ii)}$$

Combining equations (i) and (ii),

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 B_1 \\ C_1 D_1 \end{bmatrix} \begin{bmatrix} A_2 B_2 \\ C_2 D_2 \end{bmatrix} \begin{bmatrix} V_2^{'} \\ -I_2^{'} \end{bmatrix} = \begin{bmatrix} AB \\ CD \end{bmatrix} \begin{bmatrix} V_2^{'} \\ -I_2^{'} \end{bmatrix}$$

$$\text{Hence,} \quad \begin{bmatrix} AB \\ CD \end{bmatrix} = \begin{bmatrix} A_1 B_1 \\ C_1 D_1 \end{bmatrix} \begin{bmatrix} A_2 B_2 \\ C_2 D_2 \end{bmatrix} \quad \text{———————} \quad \text{(iii)}$$

Equation (iii) shows that the resultant *ABCD* matrix of the cascade connection is the product of the individual *ABCD* matrices.

6.8.2 Parallel Connection:

Figure: 6.8.2 shows two-port networks connected in parallel. In the parallel connection, the two networks have the same input voltages and the same output voltages.

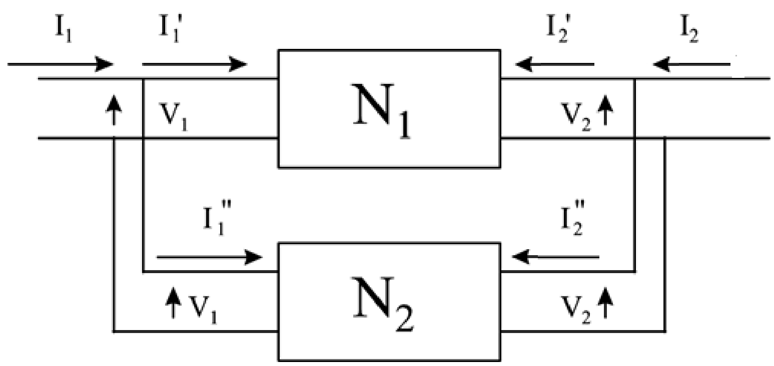


Figure: 6.8.2 Parallel Connection

Let $Y_{11}^{'}, Y_{12}^{'}, Y_{21}^{'}, Y_{22}^{'}$ be the Y-parameters of the network N_1 and $Y_{11}^{''}, Y_{12}^{''}, Y_{21}^{''}, Y_{22}^{''}$ be the Y-parameters of the network N_2.

For the network N_1, $\begin{bmatrix} I_1^{'} \\ I_2^{'} \end{bmatrix} = \begin{bmatrix} Y_{11}^{'} Y_{12}^{'} \\ Y_{21}^{'} Y_{22}^{'} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

For the networkN_2, $\begin{bmatrix} I_1^{''} \\ I_2^{''} \end{bmatrix} = \begin{bmatrix} Y_{11}^{''} Y_{12}^{''} \\ Y_{21}^{''} Y_{22}^{''} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

For the combined network, $I_1 = I_1^{'} + I_1^{''}$ and $I_2 = I_2^{'} + I_2^{''}$

$$\text{Hence,} \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1^{'} + I_1^{''} \\ I_2^{'} + I_2^{''} \end{bmatrix} = \begin{bmatrix} Y_{11}^{'} + Y_{11}^{''} Y_{12}^{'} + Y_{12}^{''} \\ Y_{21}^{'} + Y_{21}^{''} Y_{22}^{'} + Y_{22}^{''} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} Y_{12} \\ Y_{21} Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus, the resultant Y-parameter matrix for parallel connected networks is the sum of Y matrices of each individual two-port networks.

6.8.3 Series Connection:

Figure: 6.8.3 shows two-port networks connected in series. In a series connection, both the networks carry the same input current. Their output currents are also equal.

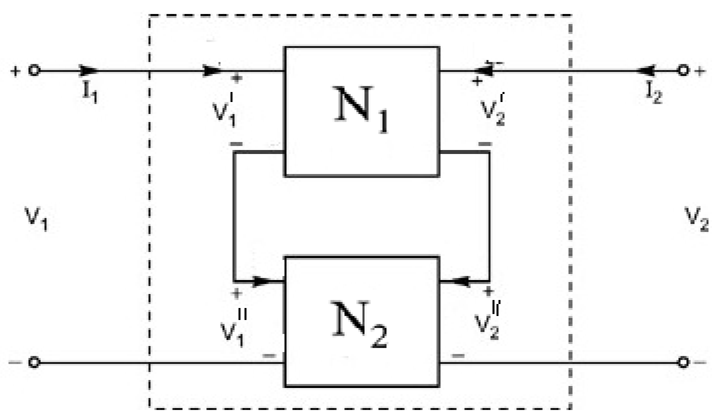


Figure: 6.8.3 Series Connection

Let $Z_{11}^{'}, Z_{12}^{'}, Z_{21}^{'}, Z_{22}^{'}$ be the Z-parameters of the network N_1 and $Z_{11}^{''}, Z_{12}^{''}, Z_{21}^{''}, Z_{22}^{''}$ be the Y-parameters of the network N_2.

For the network N_1, $$\begin{bmatrix} V_1^{'} \\ V_2^{'} \end{bmatrix} = \begin{bmatrix} Z_{11}^{'} Z_{12}^{'} \\ Z_{21}^{'} Z_{22}^{'} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the network N_2, $$\begin{bmatrix} V_1^{''} \\ V_2^{''} \end{bmatrix} = \begin{bmatrix} Z_{11}^{''} Z_{12}^{''} \\ Z_{21}^{''} Z_{22}^{''} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the combined network $V_1 = V_1^{'} + V_1^{''}$ and $V_2 = V_2^{'} + V_2^{''}$

Hence, $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1^{'} + V_1^{''} \\ V_2^{'} + V_2^{''} \end{bmatrix} = \begin{bmatrix} Z_{11}^{'} + Z_{11}^{''} Z_{12}^{'} + Z_{12}^{''} \\ Z_{21}^{'} + Z_{21}^{''} Z_{22}^{'} + Z_{22}^{''} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} Z_{12} \\ Z_{21} Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Thus, the ersultant Z-parameters matrix for the series-connected networks is the sum of Z matrices of each individual two-port network.

Assignment cum tutorial questions

SECTION-A

1. A voltage of 10V applied at port-1 results in I_1=5A and V_2=5V when port 2 is open circuited. The open circuit transfer impedance is____

   a) 2Ω b) 1Ω c) 3Ω d) 4Ω

2. For a two port network, Z11 and Z22 are equal and also Z12=Z21.Then the two port network is

a) Symmetric only b) Reciprocal but not symmetrical

c) Symmetric and Reciprocal d) Symmetrical but not reciprocal

3. The parameter Y11 in terms of Z parameters is..........

a)$\frac{Z22}{Dz}$ b)$\frac{-Z12}{Dz}$ c)$\frac{Z11}{Dz}$ d)$\frac{-Z21}{Dz}$

4. With port 1 short circuited, a voltage of 10V is applied at port 2 results in I_2=4A and I_1=-2A.The short circuit driving point admittance at port 2 is__

a) -0.4 b)0.2 c)-0.2 d)0.4

5. For a given two port network, the S/C parameters are Y_{11}=10 ,Y_{12}=Y_{21}=2 ,Y_{22}=5.The value of Z21 is___

a)$\frac{10}{23}\Omega$ b)$\frac{5}{23}\Omega$ c)$\frac{-1}{23}\Omega$ d)$\frac{1}{23}\Omega$

6. For a given two port network,Z11=10Ω;Z22=8Ω;Z12=Z21=3Ω.A resistance of 5Ω is connected at port 2.Then the driving point impedance at port 1 is__

a)$\frac{121}{3}\Omega$ b)7 Ω c)$\frac{134}{13}\Omega$ d)$\frac{108}{13}\Omega$

7. In a two port network, the parameters A=D=2 and B=3Ω.Then value of parameter C is_

a) 2 b) 1 mho c) $\frac{1}{2}$ mho d) $\frac{1}{3}$ mho

8. In a two port network, the expression for Z11 in terms of ABCD parameters is

a)$\frac{D}{C}$ b)$\frac{1}{C}$ c)$\frac{B}{C}$ d)$\frac{A}{C}$

9. For a two port symmetrical network, the relation in transmission parameters is

a) A= $\frac{1}{CD}$ b) CA=BD c) A= $\frac{1}{D}$ d) A=D

10. Two identical two port networks (having same port parameters) are connected in cascade. The parameters A of the combined network is

a) A+BC b) A2+BC c) A+ $\frac{B}{C}$ d) $\frac{A}{2}$

11. When port 1 of a two-port circuit is short-circuited, $I_1 = 4I_2$ and $V_2 = 0.25I_2$. Which of the following is true?

(a) $y_{11} = 4$ (b) $y_{12} = 16$ (c) $y_{21} = 16$ (d) $y_{22} = 0.25$

12. A two-port is described by the following equations:

$V1 = 50I_1 + 10I_2$ $V2 = 30I_1 + 20I_2$

which of the following is not true?

(a) $z_{12} = 10$ (b) $y_{12} = -0.0143$ (c) $h_{12} = 0.5$ (d) A = 50

13. If two-port is reciprocal, which of the following is not true?

(a) $z_{21} = z_{12}$ (b) $y_{21} = y_{12}$ (c) $h_{21} = h_{12}$ (d) $AD = BC + 1$

14. A passive 2 port network is in a steady-state compared to its input, the steady stay output can never offer

A. Better regulation.

B. Higher voltage.

C. Greater Power

D. Lower impedance.

15. Which elements act as an independent variables in Y-parameters?
a. Current
b. Voltage
c. Both (a&b)
d. None of the above

16. If the two ports are connected in cascade configuration, then which arithmetic operation should be performed between the individual transmission parameter in order to determine overall transmission parameters?

a. Addition   b. Subtraction   **c**. Multiplication   d. Division

SECTION-B

1. Determine the Z-parameters for the network shown in fig (a).

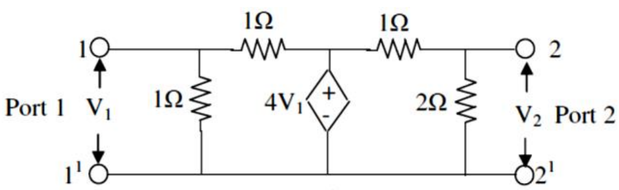


Fig.(a)

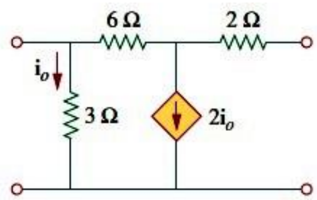
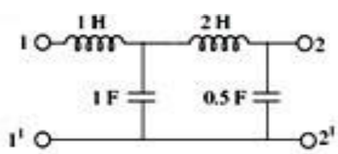


Fig.(b)

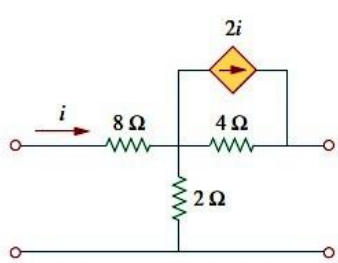
2. Obtain the y parameters for the circuit in Fig(b)

3. The y-parameters of a two port network are y_{11}=15 mho, y_{22}=24 mho, y_{12}=y_{21}=6 mho. Determine ABCD parameters.

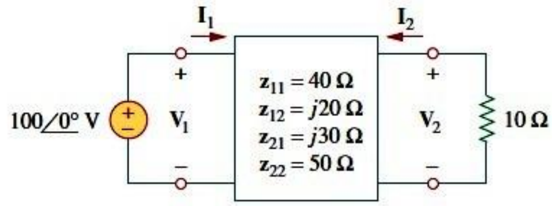
4. Determine ABCD parameters of the network shown in fig.



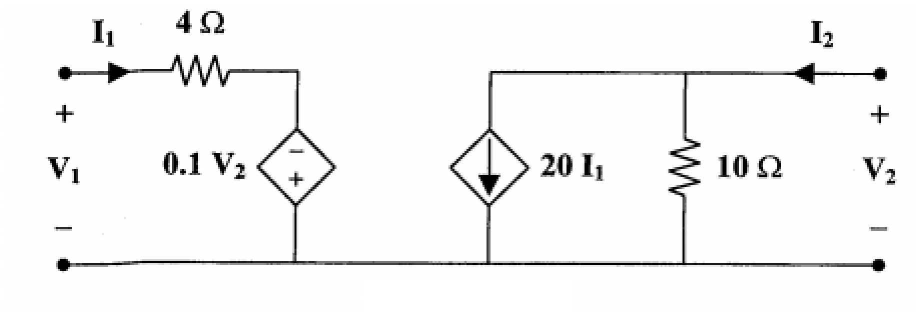
5. Determine the y parameters for the two-port shown in Fig.



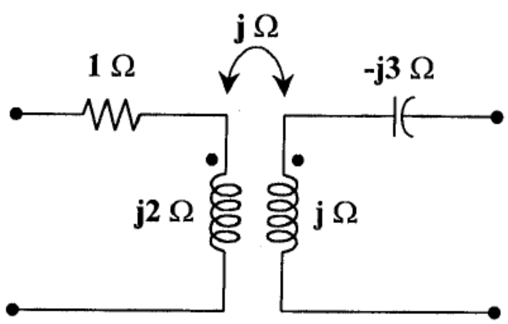
6. Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit in Fig.



7. Find the y parameters for the circuit shown below:

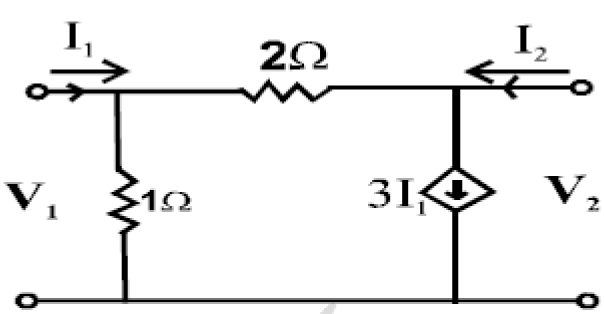


8. Obtain T-parameters for the circuit shown in the figure below



SECTION-C

1. The open circuit impedance matrix of the two-port network shown in figure is:



a) $\begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix}$ b) $\begin{bmatrix} -2 & -8 \\ 1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} -2 & -1 \\ -1 & 3 \end{bmatrix}$

2. Two two-port networks are connected in cascade. The combination is to represented as a single two-port networks. The parameters of the network are obtained by multiplying the individual:

a) z-parameter matrix b) h-parameter matrix

c) y-parameter matrix d) ABCD parameter matrix

3. For a two-port network to be reciprocal

a) $z_{11} = z_{22}$ b) $y_{21} = y_{12}$ c) $h_{12} = -h_{21}$ d) AD-BC=0

4. The condition that a z-port network is reciprocal, can be expressed in terms of its ABCD parameters as:

a) AD-BC=1 b) AD-BC=0 c) AD-BC>1 d) AD-BC<1

5. The short-circuit admittance matrix of a two-port network is:

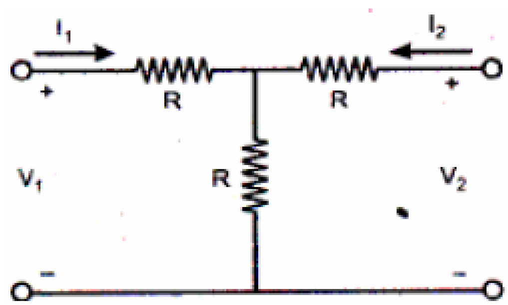
$$\begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix}$$

The two-port network is:

a) Non-reciprocal and passive b) Non-reciprocal and active

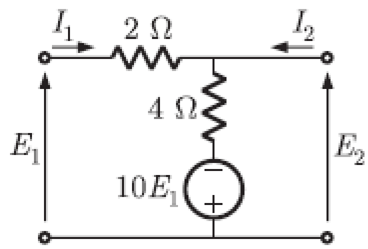
c) Reciprocal and passive d) Reciprocal and active

6. A two-port network is shown in figure. The parameter $h21$ for this network can be given by:



a) −1/2 b) +1/2 c) −3/2 d) +3/2

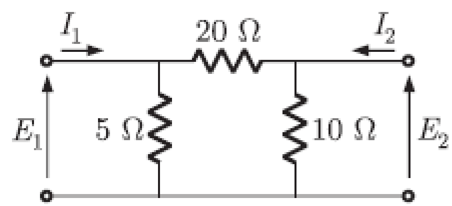
7. The Z parameters Z_{11} and Z_{21} for the 2-port network in figure are:



a) $Z_{11} = -\frac{6}{11}\Omega \quad Z_{21} = \frac{16}{11}\Omega$ b) $Z_{11} = \frac{6}{11}\Omega \quad Z_{21} = \frac{4}{11}\Omega$

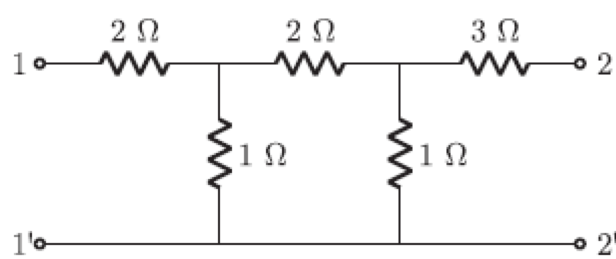
c) $Z_{11} = \frac{6}{11}\Omega \quad Z_{21} = -\frac{16}{11}\Omega$ d) $Z_{11} = \frac{4}{11}\Omega \quad Z_{21} = \frac{4}{11}\Omega$

8. The admittance parameter Y_{12} in the two-port network in figure is:



a) –0.2 mho b) 0.1 mho c) –0.05 mho d) 0.05 mho

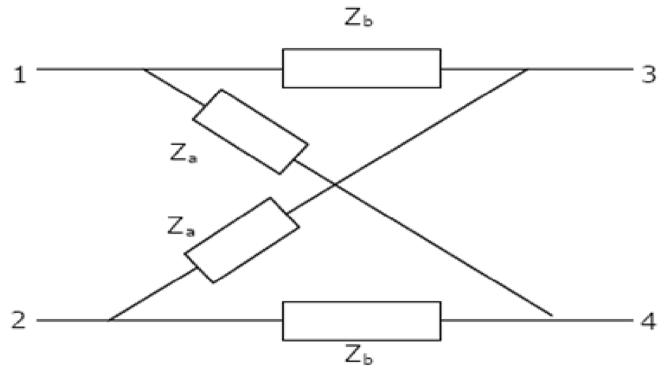
9. The impedance parameters $Z11$ and $Z12$ of the two-port network in figure are:



a) Z_{11} = 2.75 Ω and Z_{12} = 0.25 Ω b) Z_{11} = 3 Ω and Z_{12} = 0.5 Ω

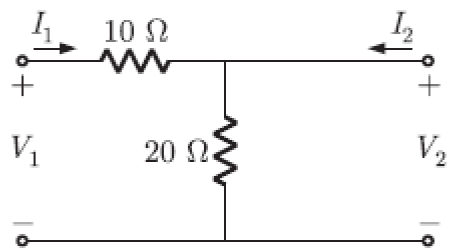
c) Z_{11} = 3 Ω and Z_{12} = 0.25 Ω d) Z_{11} = 2.25 Ω and Z_{12} = 0.5 Ω

10. For the lattice shown in figure, $Z_a = j2\Omega$ and $Z_b = 2\Omega$. The values of the open circuit impedance parameters $Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ are



a) $\begin{bmatrix} 1-j & 1+j \\ 1+j & 1+j \end{bmatrix}$ b) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$ c) $\begin{bmatrix} 1+j & 1+j \\ 1-j & 1-j \end{bmatrix}$ d) $\begin{bmatrix} 1+j & -1+j \\ -1+j & 1+j \end{bmatrix}$

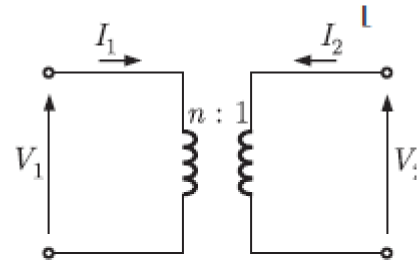
11. The h parameters of the circuit in figure are:



(a) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$

(c) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & +1 \\ -1 & 0.05 \end{bmatrix}$

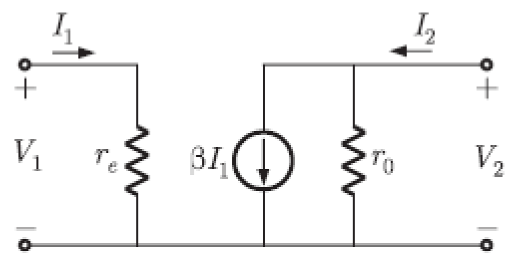
11. The $ABCD$ parameters of an ideal n:1 transformer shown in the figure are $\begin{bmatrix} n & 0 \\ 0 & x \end{bmatrix}$



The value of x will be

a) n b) $\frac{1}{n}$ c) n^2 d) $\frac{1}{n^2}$

12. In the two port network shown in the figure below, Z_{12} and Z_{21} and respectively



a) r_e and βr_0 b) 0 and $-\beta r_0$ c) 0 and βr_0 d) r_e and $-\beta r_0$

13. A two-port network is represented by *ABCD* parameters given by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

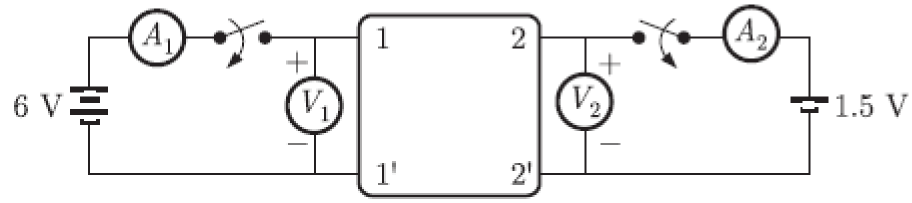
(A) $\frac{A+BR_L}{C+DR_L}$

(B) $\frac{AR_L+C}{BR_L+D}$

(C) $\frac{DR_L+A}{BR_L+C}$

(D) $\frac{B+AR_L}{D+CR_L}$

15.A two-port network shown below is excited by external DC source. The voltage and the current are measured with voltmeters V_1, $V_2$and ammeters. A_1, A_2(all assumed to be ideal), as indicated



Under following conditions, the readings obtained are:

(1) $S1$ -open, $S2$ - closed $A_1 = 0, V_1 = 4.5$ V, $V_2 = 1.5$ V, $A_2 = 1$ A

(2) $S1$ -open, $S2$ - closed $A_1 = 4$ A, $V_1 = 6$ V, $V_2 = 6$ V, $A_2 = 0$

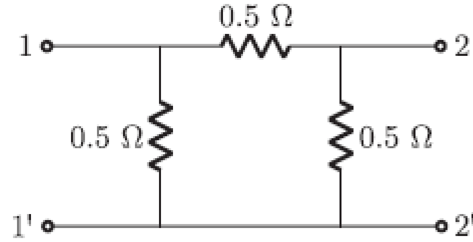
16. The z -parameter matrix for this network is

a) $\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 1.5 \end{bmatrix}$ b) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 4.5 \end{bmatrix}$ c) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$ d) $\begin{bmatrix} 4.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

17. The h-parameter matrix for this network is

a) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$ b) $\begin{bmatrix} -3 & 1 \\ 3 & 0.67 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$ d) $\begin{bmatrix} 3 & 1 \\ -3 & -0.67 \end{bmatrix}$

18. For the two-port network shown below, the short-circuit admittance parameter matrix is



(A) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$ S

(B) $\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$ S

(C) $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ S

(D) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ S

19. With 10 V dc connected at port *A* in the linear nonreciprocal two-port network shown below, the following were observed :

(i) 1 Ω connected at port *B* draws a current of 3A

(ii) 2.5 Ω connected at port *B* draws a current of 2A

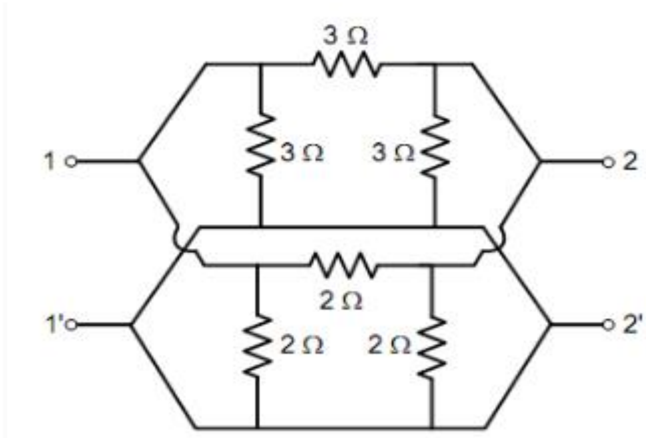
19. With 10 V dc connected at port *A*, the current drawn by 7 Ω connected at port *B* is

a) 3/7 A b) 5/7 A **c) 1A** d) 9/7 A

20. For the same network, with 6 V dc connected at port *A*, 1 Ω connected at port *B* draws 7/3 A. If 8 V dc is connected to port *A*, the open circuit voltage at port *B* is

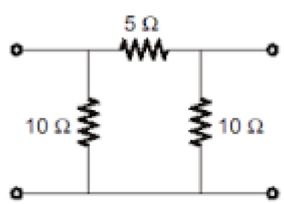
a) 6 V **b) 7 V** c) 8 V d) 9 V

21. In the h – parameter model of 2 – port network given in the figure shown,



The value of h_{22} (in Siemens) is ____________

22. The 2 – port Admittance matrix of the circuit shown is given by___________



a) $\begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$ b) $\begin{bmatrix} 15 & 5 \\ 5 & 15 \end{bmatrix}$ c) $\begin{bmatrix} 3.33 & 5 \\ 5 & 3.33 \end{bmatrix}$ d) $\begin{bmatrix} 0.3 & 0.4 \\ 0.4 & 0.3 \end{bmatrix}$